The Open Economy Macroeconomics of Central Bank Digital Currencies

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Abstract

We study the open-economy implications of introducing CBDCs into a 2-country DSGE environment that features a realistic financial system, with households deriving liquidity services from both CBDCs and bank deposits. We make several assumptions about the architecture and design features of CBDCs: 1) We focus on retail CBDC, and allow households to hold CBDCs in any currency; 2) CBDCs are strictly separated from reserves, and are remunerated at an interest rate below the policy rate due to their non-pecuniary convenience yield; 3) CBDCs are introduced via central bank purchases of government bonds or transfers to the government budget, ruling out direct and guaranteed conversion of bank deposits into CBDC at commercial banks; 4) CBDCs are separately issued in both countries. We show that the introduction of CBDCs by a single economy is highly beneficial in terms of output and welfare. The effects of financial disturbances are not exacerbated by the presence of CBDCs, in fact their effect on banks is typically mitigated. Large reallocations of liquidity between currencies, and between deposits and CBDC, yield benign balance sheet adjustments and small real effects. Finally, a more aggressively countercyclical use of the interest rate on CBDC could be highly beneficial in terms of stabilizing output and inflation.

JEL Codes: TBC

Keywords: Central bank digital currencies, monetary policy, bank deposits, bank loans, monetary frictions, money demand, money supply, credit creation.

Acknowledgements: TBC

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1. Introduction

The worldwide drive towards the creation of retail central bank digital currencies (CBDCs) is set to mark a key milestone in the evolution of money. Spurred by recent advances in technologies, more and more central banks are exploring ways to issue digital central bank money to the general public, effectively creating "reserves for all".

The microeconomic implications of CBDC have so far received most of the attention. Wider access to central bank balance sheets is currently a privilege of only a few financial institutions. By meeting the demand for a modern medium of exchange that is accessible to all households, firms and banks, CBDC promises lower transactions costs. By providing a safe and universally accessible store of value that is denominated in the national currency, CBDC could improve financial access for the unbanked population and offer unique advantages over other liquid assets, including private cryptocurrencies with their widely fluctuating exchange rates. And by lessening dependence on centralised clearing and book-keeping, a well-designed CBDC could improve the resilience of the payments system.

The macroeconomic implications of CBDC could however also be highly beneficial. Concerning the level of real activity, efficiency gains from CBDC, due to a reduction in debt, leverage and therefore interest rates, due to a reduction in distortionary taxes made possible by lower interest rates on government liabilities, and due to cheaper provision of liquidity to the economy, could translate into higher aggregate output and welfare. Concerning the stability of real activity, the ability to adjust either the remuneration or the supply of CBDC would give the central bank an additional policy tool, which could allow it to better stabilize output and inflation. Concerning the stability of the financial system, this is likely to be a function of the design of a CBDC system. For instance, some have argued that when CBDC is a close substitute for bank deposits, this increases the risks of bank disintermediation and could therefore have adverse implications for financial stability (see e.g. Agur et al. (2019)). However, there are various design features of CBDC that could much reduce this risk, see e.g. Bindseil (2020) and Kumhof and Noone (2021).

While the macroeconomic implications of CBDC have been studied in closed economy contexts in several papers now, the open-economy dimension has only been studied in very few papers. This is despite the enormous relevance of this topic to policymakers, who therefore have had to rely on preliminary, verbal and non-analytical, assessments. On the one hand, it is held that a more efficient means of cross-border payments could broaden market participation and promote better international risk sharing. On the other hand, it is feared that cross-border use of CBDCs may amplify existing financial frictions and exacerbate volatility of capital flows and exchange rates, or even introduce new vulnerabilities, such as the risk of sudden runs into or out of CBDCs. A recent survey suggests that this is indeed a concern of many central banks, not least in Emerging Market economies (Auer et al. (2021)). Other risks include potential currency substitution and loss of monetary policy independence if one CBDC becomes dominant internationally (see BIS (2021) "Report to the G20").

This paper examines the open-economy implications of CBDC in a medium-sized 2-country DSGE model featuring a rich and realistic financial system. We build on the setup of Kumhof et al. (2020), where loan issuance and deposit creation underpins gross capital flows and governs its dynamics. Given CBDCs' money-like characteristics, this framework is particularly well-suited for studying the introduction, and existence alongside deposit money, of CBDC, and its implications for capital flows and exchange rates. We therefore follow Barrdear and Kumhof (2021), by extending Kumhof et al. (2020) so that households jointly derive liquidity services from CBDC and bank deposits. We also introduce government bonds in positive net supply, and let a wholesale money market, which we label "financial investors" in the model, arbitrage between government bonds and wholesale bank deposits. This setup describes in a stylised way the likely nature of retail and wholesale money markets in a post-CBDC world: households utilise CBDC and retail bank deposits for retail purchases (consumption), financial investors maximise profit by holding higher-yielding government bonds and wholesale bank deposits, and banks create both retail and wholesale bank deposits through loans. Finally, banks provide a payments platform that allows households, financial investors, and the government to exchange bonds, CBDC, wholesale deposits, and retail deposits.

Our setup makes several important and non-trivial assumptions about the architecture and design features of CBDCs. First, we focus on the retail variety of CBDC, and we assume full cross-border interoperability that enables households (and firms, which in our model are mostly merged with households) to hold CBDC in any currency. Central banks today are indeed actively working towards a CBDC design that would ensure such interoperability. Second, CBDCs are remunerated at an interest rate that, because of CBDCs' non-pecuniary convenience yield, remains substantially below the conventional policy interest rate, the rate on interbank reserves. Reserves and CBDC are therefore strictly separate forms of central bank money, with the former exclusively held by financial institutions, and the latter almost exclusively (except to the extent that banks need to hold a CBDC float to function as market makers) by households and firms. We explore both the case where the central bank sets the CBDC interest rate and where it controls the quantity of CBDC. In each case the central bank acquires an additional monetary policy tool. Third, we assume that a central bank introduces CBDC through either purchases of government bonds, or through transfers to the government budget that finance part of government spending. Direct and guaranteed conversion of bank deposits into CBDC at commercial banks, which always requires a backup guarantee of conversion by the central bank, is ruled out. This assumption reflects current practice in the issuance of other forms of central bank money, which always takes place against eligible assets, and furthermore this does not prevent the central bank from issuing CBDC against bank deposits at its discretion and during most times. Nevertheless, the absence of guaranteed conversion rules out system-wide or aggregate runs from bank deposits into CBDCs. Fourth, we mostly consider cases where CBDCs exist in both countries, but we also study the asymmetric case where only one country issues a CBDC.

Concerning the level of real activity, expanding on the closed-economy results of Barrdear and Kumhof (2021), we show that the introduction of a retail CBDC in our model is unambiguously positive for output and welfare. In particular, CBDC issuance lowers the equilibrium real interest rate by reducing the oustanding stock of defaultable government debt (CBDC is non-defaultable, as is not debt, see Kumhof et al. (2021)), it permits the government to lower distortionary taxes because the interest carrying cost of remaining defaultable government debt declines while the carrying cost of CBDC is even lower, and it allows the government to provide more abundant and cheaper liquidity than what banks alone are able to do. The open-economy implications of the asymmetric introduction of CBDC by one economy also include a small long-run depreciation and a small long-run current account deficit.

Concerning the volatility of real and financial activity, we are able to assess whether CBDCs are likely to exacerbate the macroeconomic, capital flow, and exchange rate effects of real and financial disturbances, such as sudden portfolio preference shifts into an economy's currency or more speficially into an economy's CBDC. Our assessment is that the consequences of CBDC are mostly benign. Even in an economy with CBDC, the real implications of large demand-driven reallocations of liquidity from one currency into another appear to be modest. And various combinations of liquidity preference shifts between bank deposits and CBDCs denominated in both currencies typically also yield benign gross balance sheet adjustments and small real effects.

Finally, we study the consequences of the central bank having access to a second policy tool in the presence of CBDC. We find that, for any given interest rate rule for the interest rate on reserves, a more aggressively countercyclical use of the interest rate on CBDC could be highly beneficial, both by increasing the countercyclical effects of the interest rate on reserves and by contributing to stabilizing financial and real activity in its own right, specifically by withdrawing liquidity and therefore purchasing power in a boom and vice versa in a bust. Importantly, this requires that in a boom the interest rate on CBDC should be lowered, not increased, and lowered relative to the policy rate and not necessarily (but possibly) in absolute terms.

The rest of the paper is organized as follows. Section 2 briefly studies the related literature. Section 3 presents a sketch of the model, with details relegated to a technical appendix. Section 4 discussed calibration. Section 5 presents the results of our model simulations. Section 6 concludes.

2. Literature Review (tbc)

Similarly to our paper, Ferrari et al. (2020) study CBDCs in a two-country DSGE setup. They find that the presence of CBDC strengthens international linkages by serving as an additional asset for cross-border arbitrage that accentuates the responses of exchange rates to shocks. As in our model, CBDC provides liquidity services, is interest-bearing and can be held both domestically and by foreigners. Unlike our model, their CBDC is a substitute in liquidity services only for cash and not for bank deposits. This is a crucial distinction, because the explicit modeling of the payment system interactions of CBDCs and bank deposits allows us to study the financial sector implications of the introduction of, and shocks to the demand for, CBDCs, and the ensuing policy issues.

Closer to our approach is George et al. (2020), who extend the framework of Barrdear and Kumhof (2021) to a small open economy. They study the welfare properties of a CBDC economy, and find that when the central bank uses CBDC as an active monetary policy tool and follows a quantity rule, welfare is highest. Our setup is more general and nests the small open economy as a special case. This allows us to focus on cross-border issues such as capital flows and exchange rate volatility in general equilibrium, including the important case where more than one country issues a CBDC.

3. The Model

3.1. Overview

The model setup mostly follows that of Kumhof et al. (2020), with a few important exceptions that are adapted from the closed-economy framework of Barrdear and Kumhof (2021). First, capital rather than land plays the roles of the second factor of production and of the asset used as collateral. Second, the presence of CBDC requires a richer specification of the household demand for liquidity that goes beyond bank deposits, and it also requires the introduction of the financial investor sector. Third, for the purpose of studying the effiency effects of CBDC we introduce a fiscal block with a full set of distortionary taxes.

The world economy consists of two countries, Home and Foreign, with respective shares in the world population of n and $1-n$. Each country is populated by households, financial investors, manufacturers, unions, banks and a government. In our calibration $n = 0.5$ and the two countries are fully symmetric in economic structure and in the calibration of all parameters. Households in each country own the domestic capital stock, which serves both as an input into the production function of domestic value added and as collateral for borrowing from banks, and households make real investment decisions that affect the rate of accumulation of capital. Household income consists of capital rentals, net investment income from their financial holdings of bank deposits and CBDCs, wages, lump-sum profit distributions from manufacturers, unions and banks, and net taxes and transfers. Households, both domestic and foreign, are the only retail borrowers from and retail depositors at banks, and are the sole holders of retail CBDCs. They consume a CES composite of domestic and foreign goods, and they purchase these goods using a CES composite of domestic and foreign[1] currency liquidity. Liquidity in each currency in turn consists of bank deposits, which are created by banks through loans, and CBDCs, which are created by central banks through purchases of government bonds or transfers to the government budget.

Financial investors represent a share $\varpi = 0.05$ of the population in each country. While capital, retail bank deposits and CBDC are solely held by households, domestic government bonds and wholesale bank deposits are soley held by financial investors. The economic role of financial investors can be thought of as that of market makers in wholesale money markets, which trade government bonds and wholesale bank deposits. Manufacturers and unions have pricing power, and set prices and wages subject to nominal rigidities. Monetary policy targets inflation by setting the risk-free interest rate on reserves (which are not explicitly modelled) following an inflation forecast-based rule. Once a CBDC is issued, monetary policy also controls either the interest rate on or the quantity of CBDC through a second policy rule. The government issues debt, taxes and spends, with appropriate fiscal rules ensuring the stability of the deficit-to-GDP ratio and of government spending as a share of GDP, and the extent to which different taxes are adjusted in response to permanent or temporary changes in net interest expenses.

The banking sector has three functions. The first is wholesale lending and deposit issuance, with an optimal choice of the overall balance sheet size and composition in order to maximize net worth. The second and third are retail lending and deposit issuance, with an optimal choice of the terms of loan and deposit contracts. Banks' net worth maximization is subject to minimum

[1] Note that the household sector implicitly includes firms. Firms use foreign currency deposits to pay for imports and to receive export revenues.

capital adequacy regulation (MCAR), which imposes penalties on banks whose capital drops below a specified minimum percentage of total assets, foreign currency monetary transactions costs (MONFX), which requires banks to maintain correspondent accounts with foreign banks in order to compensate for the absence of a lender of last resort in foreign currency, and foreign exchange mismatch rules (FXMR), which describe either prudential rules or banks' policies regarding the matching of balance sheet exposures to foreign account holders. The specification of bank lending in the model is based on the costly state verification model of Bernanke et al. (1999). This is modified to allow for non-contingent lending rates, and therefore for ex-post bank losses and a role for loss-absorbing capital and capital regulation. It is also modified to allow for time-varying shares of collateral that banks take into account when assessing the creditworthiness of potential borrowers. The specification of the medium of exchange function of liquid assets in the model is based on the Schmitt-Grohé and Uribe (2004) transactions cost technology, which includes bank deposits and CBDCs as imperfect substitutes.

Home banks issue loans exclusively in Home currency in order to create deposits in Home currency, and Foreign banks only issue loans in Foreign currency to create deposits in Foreign currency.[2] Equally, each central bank can issue a CBDC denominated in its country's currency. Because households require positive quantities of both currencies, they must bank with banks in both countries, and once they are introduced, also hold positive quantities of both CBDCs. Thus, their loan and deposit exposures to banks in the foreign country, as well as their cross-border CBDC holdings, are part of the economy's gross and net foreign asset positions, with the remainder accounted for by interbank loan and deposit exposures. A key implication of this set-up is that relative currency demands and supplies become an important determinant of exchange rates, alongside relative goods demands and supplies and standard interest parity conditions. This aspect is treated in much greater detail by Cesa-Bianchi et al. (2019), with will be briefly reprised below.

In the model there are three links between Home and Foreign. First, households and financial investors in both countries trade Home and Foreign goods. Second, households and banks in each country maintain cross-border balance sheet positions in both Home and Foreign currencies. Third, households in each country are permitted to hold CBDCs issued in the other country and currency. Figure 1 shows the economy's main financial and real cross-border flows.

3.2. Conventions and Assumptions

Except where specifically mentioned, our model description limits itself to the Home economy. Where interactions with Foreign are described, superscript asterisks * indicate Foreign variables. We observe the convention that a real normalized variable is the nominal variable divided by the price level P_t and the level of global productivity T_t. The exogenous and constant growth rate of global productivity is $x = T_t/T_{t-1}$, while the endogenous and time-varying growth rate of the price level is $\pi_t^p = P_t/P_{t-1}$. The nominal exchange rate E_t is the price, expressed in domestic currency, of a unit of foreign currency (so that an increase indicates a depreciation of the domestic currency), and its depreciation rate is defined as $\varepsilon_t = E_t/E_{t-1}$. The real exchange rate is defined as the ratio of the two countries' CPI price levels expressed in a common currency, $e_t = (E_t P_t^*)/P_t$.

[2] Cesa-Bianchi et al. (2019) consider the case where banks in each country issue loans in both currencies, but exclusively to households in their respective countries. Their setup only gives rise to cross-border financial positions between banks, and is therefore well-suited to study monetary aspects of exchange rate determination, but not many of the capital flow questions addressed in the present paper.

Nominal variables are denoted by upper case letters, real variables are denoted by the corresponding lower case letters (for loans, the symbols are L and ℓ), and real normalized variables are denoted by the symbol for the corresponding real variable with a check symbol above the variable. The real value of Home/Foreign currency assets is always expressed in terms of Home/Foreign goods, irrespective of whether the holder is located in Home or Foreign. Home and Foreign goods production and consumption and Home and Foreign currency balance sheet positions are indicated by the subscripts H and F, with a shorthand X for $X \in \{H, F\}$. Superscripts h, f and b indicate balance sheet positions of Home households, Foreign households and banks, with a shorthand x for $x \in \{h, f, b\}$. For the example of the domestic real value of domestic and foreign currency deposits held by Home households we therefore have $\check{d}^h_{H,t} = d^h_{H,t}/T_t = D^h_{H,t}/\left(T_t P_t\right)$ and $e_t \check{d}^h_{F,t} = e_t d^h_{F,t}/T_t = \left(\left(E_t P^*_t\right)/P_t\right)\left(D^h_{F,t}/\left(T_t P^*_t\right)\right)$, respectively.

All interest rates are in gross terms, and a subscript t on a nominal interest rate denotes an interest rate paid on an asset held from period t to period $t+1$. The real interest rate on a generic domestic currency balance sheet item Z in Home is given by $r_{zH,t} = i_{zH,t-1}/\pi^p_t$, while the real interest rate on a generic foreign currency balance sheet item Z is given by $r_{zF,t} = \left(i_{zF,t-1}\varepsilon_t\right)/\pi^p_t$. We generally describe original optimization problems in nominal and agent-specific form, while optimality conditions are shown in real, normalized and aggregate form.

3.3. Households

3.3.1. Lifetime Utility and Constraints

Households account for $1-\varpi = 0.95$ of the population mass and are indexed by j. Households maximize lifetime utility subject to sequences of intertemporal budget constraints and bank participation constraints, by choosing plans for consumption $c_t(j)$, hours worked $h_t(j)$, business investment $I_t(j)$, loans in both currencies $L^h_{X,t}(j)$, deposits in both currencies $D^h_{X,t}(j)$, CBDC holdings in both currencies $M^h_{X,t}(j)$ and capital holdings $k_t(j)$. Their consumption bundle is a CES physical aggregate that includes both domestic and foreign goods $c_{H,t}(j)$ and $c_{F,t}(j)$. They face monetary transaction costs for purchases of the consumption bundle that are decreasing in a CES liquidity aggregate that includes the purchasing power of both domestic and foreign currency liquidity, $O^h_{H,t}(j)$ and $O^h_{F,t}(j)$, which are in turn aggregates of bank deposits and CBDCs in the respective currency. The objective function for domestic household j is

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t_{hh} \left\{ S^c_t \left(1 - \frac{\nu}{x}\right) \log\left(c^{hh}_t(j) - \nu c^{hh}_{t-1}\right) - \psi_{hh} \frac{h^{hh}_t(j)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} , \tag{1}$$

where β_{hh} is the the discount factor, ν is external habit persistence, η is the elasticity of labour supply, ψ_{hh} is a labour supply scale parameter, and S^c_t is a first-order autoregressive stochastic process for consumption preferences. The CES consumption bundle over Home and Foreign goods, with consumption home-bias parameter b^c and elasticity of substitution θ_c, is

$$c^{hh}_t(j) = \left[\left(b^c\right)^{1/\theta_c} \left(c^{hh}_{H,t}(j)\right)^{\frac{\theta_c - 1}{\theta_c}} + \left(1 - b^c\right)^{1/\theta_c} \left(c^{hh}_{F,t}(j)\right)^{\frac{\theta_c-1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}} , \tag{2}$$

with corresponding utility-based price index P_t. The Home and Foreign goods sub-aggregates are in turn given by CES bundles over a continuum of goods, with elasticities of substitution θ_p. We make the conventional assumption that $\theta_p > \theta_c$. Demand functions for Home and Foreign goods are standard. Households demand a real liquidity aggregate $liq_t(j)$ that consists of the purchasing power of domestic and foreign currency deposits and CBDCs, and that reduces the monetary transactions costs of purchasing the bundle of consumption goods. The functional form for transactions costs is a simplified version of Schmitt-Grohé and Uribe (2004),

$$s_t^c(j) = A^c S_t^{md} \left(v_t^c(j)\right)^{\mathfrak{m}} \quad , \quad v_t^c(j) = \frac{4c_t^{hh}(j)\left(1+\tau_{c,t}\right)}{liq_t(j)} \ , \tag{3}$$

where the parameter A^c determines steady state velocity, and thereby the size of money demand and of banks' balance sheet, $v_t^c(j)$ is the endogenous velocity of circulation of money, $\mathfrak{m}$ determines the elasticity of monetary transactions costs with respect to velocity, $\tau_{c,t}$ is the consumption tax rate, and S_t^{md} is a first-order autoregressive shock to liquidity preferences. We will think of the latter as "flight to safety" shocks, because an increase in S_t^{md} can be thought of as an increase in the demand for the safety of money, in either currency, at a given level of real activity. The CES liquidity aggregate has a top level that combines domestic and foreign currency, and a bottom level that combines bank deposits and CBDC in the respective currencies.

For the top level liquidity aggregate, the "financial home bias" parameter is b^o and the elasticity of substitution is θ_o. In nominal terms, we have

$$LIQ_t(j) = \left[\left(b^o S_t^{mm}\right)^{1/\theta_o} \left(O_{H,t}^h(j)\right)^{\frac{\theta_o-1}{\theta_o}} + \left(1 - b^o S_t^{mm}\right)^{1/\theta_o} \left(O_{F,t}^h(j)\right)^{\frac{\theta_o-1}{\theta_o}} \right]^{\frac{\theta_o}{\theta_o-1}} \ , \tag{4}$$

where S_t^{mm} is a first-order autoregressive stochastic process for the demand for foreign versus domestic currency. We will think of shocks to $S_t^{mm^*}$ as "flight to the dollar" shocks, because a decrease in $S_t^{mm^*}$ can be thought of representing an increase in Foreign households' demand for Home currency ("dollars") relative to Foreign currency, at a given level of real activity. In equilibrium the main effect of a decrease in $S_t^{mm^*}$ will be a depreciation of the foreign currency, with the size of the depreciation depending on the substitutability θ_o between domestic and foreign currencies. The derivatives of real liquidity with respect to its two arguments are denoted by $liq_t^{H'}(j)$ and $liq_t^{F'}(j)$.

For the two bottom-level liquidity aggregates, which we will henceforth describe as different versions of the liquidity generating function or LGF, we distinguish between a separable and a nonseparable CES version. The separable version is used for the transition simulation from a pre-CBDC economy to a CBDC economy, to take account of the fact that a nonseparable aggregate would imply that before the introduction of CBDC aggregate liquidity equalled zero. In nominal terms, we have

$$\begin{aligned} O_{H,t}^h(j) &= \left(T_t P_t\right)^{1/\theta_d} \left(\left(D_{H,t}^h(j)\right)^{\frac{\theta_d-1}{\theta_d}} + \left(\eth M_{H,t}^h(j)\right)^{\frac{\theta_d-1}{\theta_d}} \right) \ , \\ O_{F,t}^h(j) &= \left(T_t P_t\right)^{1/\theta_d} \left(\left(E_t D_{F,t}^h(j)\right)^{\frac{\theta_d-1}{\theta_d}} + \left(\eth^* E_t M_{F,t}^h(j)\right)^{\frac{\theta_d-1}{\theta_d}} \right) \ . \end{aligned} \tag{5}$$

These are CES aggregates, in that the elasticity of substitution between deposits and CBDC is constant and equals θ_d. But they exhibit decreasing returns to scale in liquidity, as $\left(\theta_d - 1\right)/\theta_d < 1$. The presence of the factor $\left(T_t P_t\right)^{1/\theta_d}$ therefore ensures that this functional form remains consistent with balanced growth. The constant $\eth$ represents the technological superiority (if $\eth > 1$) of CBDC over deposits in payment transactions. The nonseparable version of the bottom level aggregate is used for all stochastic simulations. In nominal terms, they are given by

$$O_{H,t}^{h}(j) = \left[\left(b_{H}^{m}S_{H,t}^{mm}\right)^{1/\theta_d}\left(D_{H,t}^{h}(j)\right)^{\frac{\theta_d-1}{\theta_d}} + \left(1-b_{H}^{m}S_{H,t}^{mm}\right)^{1/\theta_d}\left(\eth M_{H,t}^{h}(j)\right)^{\frac{\theta_d-1}{\theta_d}}\right]^{\frac{\theta_d}{\theta_d-1}} \tag{6}$$

$$O_{F,t}^{h}(j) = \left[\left(b_{F}^{m}S_{F,t}^{mm}\right)^{1/\theta_d}\left(E_t D_{F,t}^{h}(j)\right)^{\frac{\theta_d-1}{\theta_d}} + \left(1-b_{F}^{m}S_{F,t}^{mm}\right)^{1/\theta_d}\left(\eth^{*} E_t M_{F,t}^{h}(j)\right)^{\frac{\theta_d-1}{\theta_d}}\right]^{\frac{\theta_d}{\theta_d-1}}$$

where $S_{X,t}^{mm}$ are first-order autoregressive stochastic processes for the demand for bank deposits relative to CBDC. We will use various combinations of these shocks within Home and Foreign households' liquidity aggregates to simulate runs into our out of CBDCs in one or both currencies, since this is the kind of preference shift that policmakers may be concerned about. We note that in the nonseparable case the constant $\eth$ loses the meaning of technological superiority of CBDC over bank deposits, because in this case the weight of CBDC in liquidity is determined by a combination of $\eth$ and the CES preference weights. The derivatives of these aggregates with respect to their arguments are denoted by $o_{H,t}^{D'}$, $o_{H,t}^{M'}$, $o_{F,t}^{D'}$, and $o_{F,t}^{M'}$.

Capital accumulation is given by $k_t(j) = (1-\Delta)\,k_{t-1}(j) + I_t(j)$, and the representative household's nominal flow budget constraint is[3]

$$\begin{aligned}
&\left(D_{H,t}^{h}(j) + M_{H,t}^{h}(j)\right)\left(1+\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)\right) + E_t\left(D_{F,t}^{h}(j) + M_{F,t}^{h}(j)\right)\left(1+\phi_f^{*}\left(b_t^{rat*}-\bar{b}_{ss}^{rat*}\right)\right) \quad (7)\\
&+Q_t k_t(j) - P_t\Psi_t(j) - L_{H,t}^{h} - E_t L_{F,t}^{h}(j)\\
=\ & i_{dH,t-1}^{h}D_{H,t-1}^{h}(j) + i_{mH,t-1}^{h}M_{H,t-1}^{h}(j) + E_t i_{dF,t-1}^{h}D_{F,t-1}^{h}(j) + E_t i_{mF,t-1}^{h}M_{F,t-1}^{h}(j)\\
&+Ret_{k,t}Q_{t-1}k_{t-1}(j)\left(1-\kappa_{H,t-1}^{h}S_{t-1}^{k}\Gamma_{H,t}^{h}(j) - \kappa_{F,t-1}^{h^*}S_{t-1}^{k^*}\Gamma_{F,t}^{h^*}(j)\right)\\
&-P_t\left(1+s_t^{c}(j)\right)c_t^{hh}(j)\left(1+\tau_{c,t}\right) + W_t^{wo}h_t^{hh}(j)\left(1-\tau_{L,t}\right) + P_t\frac{1-\iota}{1-\varpi}\Upsilon_t(j)\\
&+Q_t I_t(j) - P_{H,t}I_t(j) - \frac{\phi_i}{2}P_{H,t}I_t\left(S_t^{i}\frac{\left(I_t(j)\right)}{xI_{t-1}(j)}-1\right)^2 .
\end{aligned}$$

The left-hand side shows households' net asset investments and its gross components, namely capital plus deposits and CBDC in each currency minus loans in each currency, at time t. The term $\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)$, where $b_t^{rat} = B_t/\left(4GDP_t\right)$ is the government debt-to-GDP ratio and $\bar{b}_{ss}^{rat}$ is its initial steady state, represents transactions costs related to the holding of domestic financial assets, with a similar term for foreign financial assets. This cost is taken as exogenous by households, and $P_t\Psi_t(j)$ represents the lump-sum distribution of this cost back to households. The budgetary effect is therefore neutral, while marginal conditions are affected. This feature allows the model to replicate the small but positive elasticity of equilibrium real interest rates with respect to government debt found in the empirical literature. Interest rates on all financial assets in a given currency are assumed to be affected in the same fashion, so that a change in the government debt-to-GDP ratio, ceteris paribus, will affect the level of interest rates but not the structure of spreads.

The left-hand side shows why the assumption of a representative household that both borrows from and holds deposits at banks is crucial for the modeling of gross positions. Domestic gross

[3] For ease of exposition, we omit numerically very small loan adjustment cost terms that are rebated to households in a lump-sum fashion.

positions, with domestic banks, are given by $D^h_{H,t}(j) - L^h_{H,t}(j)$, and this shows that the creation of additional deposit liquidity does not require any physical saving by households, it only requires credit creation by banks, which to a first approximation has nothing at all to do with household saving. Foreign gross positions, with foreign banks, are given by $E_t\left(D^h_{F,t}(j) - L^h_{F,t}(j)\right)$, and the same comments apply. Credit creation for a domestic resident by a foreign bank therefore represents one of the most elementary forms of gross capital international financial (i.e. not involving goods trade) capital flows, which always have two inseparable legs, a gross inflow and a gross outflow, with no implications, to a first approximation, for the current account and the net foreign asset position. The presence of CBDC makes this picture more complex, but does not change the logic.

The first and second line on the right-hand side shows the gross returns on assets held in the previous period. Capital yields a nominal post-tax return $Ret_{k,t} = \left[(1-\Delta)Q_t + R^k_t - \tau_{k,t}\left(R^k_t - \Delta Q_t\right)\right]/Q_{t-1}$ that includes market price appreciation, physical depreciation, physical rentals, and taxing of physical rentals net of physical depreciation, with the real return equal to $ret_{k,t} = Ret_{k,t}/(x\pi^p_t)$. Domestic and foreign borrowers are subject to idiosyncratic productivity shocks $\omega^x_{H,t+1}$, $x \in \{h, f\}$, that are log-normally distributed with mean 1 and variance $(\sigma^x_H)^2$. We denote the pdf and cdf of these shocks by $f^x(\omega^x_{H,t+1})$ and $F^x(\omega^x_{H,t+1})$ and the cutoff productivity shocks below which bankruptcy occurs ex-post by $\bar{\omega}^x_{H,t}$, and we define $f^x_{H,t} = f^x(\bar{\omega}^x_{H,t})$ and $F^x_{H,t} = F^x(\bar{\omega}^x_{H,t})$. Households' return to capital excludes returns that go to banks to repay loans, where $S^k_t\kappa^h_{H,t-1}$ and $S^{k^*}_t\kappa^{h^*}_{F,t-1}$ are the time-varying shares of domestic capital accepted as collateral from domestic households by domestic banks (for domestic currency credit) and by foreign banks (for foreign currency credit), and where $\Gamma^h_{H,t}(j)$ and $\Gamma^{h^*}_{F,t}(j)$ are the endogenous shares of the gross returns of collateral capital that go to the lenders to repay loans. The term S^k_t is a first-order autoregressive shock to the willingness of domestic banks to lend to both domestic and foreign households.[4] An increase in S^k_t leads to a credit boom that both increases the quantity of domestic credit and liquidity creation and that reduces the cost, measured in terms of spreads, of obtaining that credit.

Expenditures consist of consumption purchases, including consumption taxes at the rate $\tau_{c,t}$ and monetary transaction costs at the rate s^c_t, and investment purchases including adjustment costs. Consumption purchases represent a CES aggregate of Home and Foreign goods (2), while investment purchases use exclusively Home goods. Income consists of labor income, at the wage rate W^{wo}_t and net of labor income taxes at the rate $\tau_{L,t}$, , plus the market value of investment goods, where Q_t is Tobin's q, plus lump-sum net income $P_t\Upsilon_t(j)(1-\iota)/(1-\varpi)$, where $(1-\iota)$ is the share of households in aggregate lump-sum net income (the remainder goes to financial investors). Lump-sum net income equals the sum of profits and dividends of manufacturers, unions and banks, price, wage, and investment adjustment costs, costs of monitoring manufacturers, penalty costs paid by banks, the share $1-\mathfrak{r}$ of monetary transactions costs related to retail and interbank deposits that is not treated as a sunk resource cost, as well as the net balance of fiscal transfers and lump-sum taxes. The nominal Lagrange multiplier of the budget constraint is denoted by $\Lambda^{hh}_t(j)$.

Households also face participation constraints for taking out loans in domestic and foreign currency, whose derivation will be detailed below. We have

$$\begin{aligned}
\mathbb{E}_t\left[\kappa^h_{H,t}S^k_t Ret_{k,t+1}Q_t k_t(j)\left(\Gamma^h_{H,t+1}(j) - \xi^h_H G^h_{H,t+1}(j)\right) - i^h_{\ell H,t}L^h_{H,t}(j)\right] &= 0\,, \qquad (8)\\
\mathbb{E}_t\left[\kappa^{h^*}_{F,t}S^{k^*}_t E^*_{t+1} Ret_{k,t+1}Q_t k_t(j)\left(\Gamma^{h^*}_{F,t+1}(j) - \xi^{h^*}_F G^{h^*}_{F,t+1}(j)\right) - i^{h^*}_{\ell F,t}L^h_{F,t}(j)\right] &= 0\,,
\end{aligned}$$

[4] An alternative to the willingness to lend interpretation is to see the κ parameters as regulatory parameters that affect permissible loan-to-value ratios.

with multipliers $\Lambda_t^{hh}\tilde{\Lambda}_{H,t+1}^h$ and $\Lambda_t^{hh}\tilde{\Lambda}_{F,t+1}^h$. The second participation constraint belongs to foreign banks, because these are the source of foreign currency credit. Here the terms $\Gamma_{X,t+1}^x(j)$ denote lenders' gross shares in pledged gross returns to capital, and $\xi_X^x G_{X,t+1}^x(j)$ denotes lenders' monitoring costs share in pledged gross returns to capital.

3.3.2. Optimality Conditions

We assume that each household holds identical initial stocks of all physical and financial assets and liabilities, and receives identical net lump-sum transfers. Because all households face the same market prices, this implies that they remain symmetric at all times. The index j can therefore be dropped when stating the optimality conditions, which are presented in real normalized form. The first-order conditions for domestic and foreign consumption goods, hours worked, capital, and investment are largely standard[5] for models in this class, and are omitted to conserve space. The first-order condition for consumption is

$$\frac{S_t^c\left(1-\frac{\nu}{x}\right)}{\check{c}_t^{hh}-\frac{\nu}{x}\check{c}_{t-1}^{hh}} = \check{\lambda}_t^{hh}\left(1+\tau_{c,t}\right)\left(1+s_t^c+s_t^{c'}v_t^c\right) . \tag{9}$$

The term $1+s_t^c+s_t^{c'}v_t^c$, which we will refer to as the effective price of consumption, exceeds 1 due to the presence of transactions costs, which can be reduced through additional liquidity provision by either banks or, in the case of CBDC, the central bank. This will play an important part in the transmission mechanism of money demand and money supply shocks.

The first-order conditions for domestic and foreign currency deposits are

$$\begin{aligned} 1+\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)-\frac{1}{4}s_t^{c'}\left(v_t^c\right)^2 liq_t^{H'}o_{H,t}^{D'} &= \mathbb{E}_t\frac{\beta_{hh}}{x}\frac{\check{\lambda}_{t+1}^{hh}}{\check{\lambda}_t^{hh}}r_{dH,t+1}^h , \\ 1+\phi_f^*\left(b_t^{rat^*}-\bar{b}_{ss}^{rat^*}\right)-\frac{1}{4}s_t^{c'}\left(v_t^c\right)^2 liq_t^{F'}o_{F,t}^{D'} &= \mathbb{E}_t\frac{\beta_{hh}}{x}\frac{\check{\lambda}_{t+1}^{hh}}{\check{\lambda}_t^{hh}}r_{dF,t+1}^h . \end{aligned} \tag{10}$$

This states that the products of the intertemporal marginal rate of substitution and the deposit interest rates are less than one, due to monetary transactions costs that are inversely related to the quantity of liquidity. Therefore, as in Cesa-Bianchi et al. (2019) and Kumhof et al. (2020), the presence of monetary transaction costs implies systematic deviations from uncovered interest parity. Combining the two first-order conditions for deposits gives an expression for what we call the MUIP spread, which captures the effects on interest parity and exchange rates of changes to the relative convenience yield of the two currencies. This convenience yields can change following changes to relative supplies or demands of currencies, due to the imperfect substitutability of the liquidity composites that enter (4).

Similarly, the first-order conditions for domestic and foreign currency CBDCs are given by

$$\begin{aligned} 1+\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)-\frac{1}{4}s_t^{c'}\left(v_t^c\right)^2 liq_t^{H'}o_{H,t}^{M'} &= \mathbb{E}_t\frac{\beta_{hh}}{x}\frac{\check{\lambda}_{t+1}^{hh}}{\check{\lambda}_t^{hh}}r_{mH,t+1} , \\ 1+\phi_f^*\left(b_t^{rat^*}-\bar{b}_{ss}^{rat^*}\right)-\frac{1}{4}s_t^{c'}\left(v_t^c\right)^2 liq_t^{F'}o_{F,t}^{M'} &= \mathbb{E}_t\frac{\beta_{hh}}{x}\frac{\check{\lambda}_{t+1}^{hh}}{\check{\lambda}_t^{hh}}r_{mF,t+1} , \end{aligned} \tag{11}$$

[5] The first-order condition for capital has several additional terms related to the use of capital as credit collateral.

where r_{mH} and r_{mF} are the real interest rates on domestic and foreign currency CBDC. They could be policy- or market-determined, depending on the form of the monetary policy CBDC rule (see below).

Finally, the first-order conditions for Home and Foreign currency loans are[6]

$$\begin{aligned} 1 &= \mathbb{E}_t \frac{\beta_{hh}}{x} \frac{\check{\lambda}^{hh}_{t+1}}{\check{\lambda}^{hh}_t} \tilde{\lambda}^h_{H,t+1} r^h_{\ell H,t+1} \;, \qquad (12) \\ 1 &= \mathbb{E}_t \frac{\beta_{hh}}{x} \frac{\check{\lambda}^{hh}_{t+1}}{\check{\lambda}^{hh}_t} \tilde{\lambda}^{h*}_{F,t+1} r^h_{\ell F,t+1} \;, \end{aligned}$$

where $r^h_{\ell H}$ and $r^h_{\ell F}$ are the real wholesale lending rates charged by domestic and foreign banks.

3.4. Financial Investors

Financial investors account for $\varpi = 0.05$ of the population mass and are also indexed by j. Their role in the model is to represent the wholesale money markets, which trade and arbitrage in government bonds and wholesale bank deposits that exhibit very high interest sensitivity. They therefore differ from households in the specification of their lifetime utility function and in the menu of assets they hold, which are restricted to these two asset classes. They are the sole investors in domestic government debt, and they make a market in wholesale deposits by being both a taker and maker of these deposits, at identical interest rates due to zero transactions costs. They also function as brokers, but not as holders, for the retail CBDC issued by the domestic central bank. The reason why they are not holders is that their asset preferences are specified to be highly interest sensitive, and retail CBDC pays a far lower interest rate than wholesale bank deposits.[7] The reason why they are needed as brokers is that central banks only issue CBDC against government bonds, and financial investors are the only holder of such bonds. However, after obtaining CBDC for their customers, households, they then immediately exchange the CBDC against bank deposits, at market clearing interest rates.

The objective function for financial investor j is

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t_{fi} \left\{ S^c_t (1 - \frac{v}{x}) \log(c^{fi}_t(j) - \nu c^{fi}_{t-1}) - \psi_{fi} \frac{h^{fi}_t(j)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \gamma \frac{\left(\frac{d^{fi}_t(j)}{T_t}\right)^{1-\frac{1}{\vartheta}}}{1-\frac{1}{\vartheta}} \right\} \qquad (13)$$

where v and η have the same values as for households. Because our objective is to represent the highly interest-sensitive nature of wholesale money markets, a deposits-in-the-utility-function term replaces the monetary transaction cost friction of households, because the latter invariably produces a money demand that is highly sensitive to real activity but much less sensitive to interest rates.

[6] We omit very small loan adjustment costs that are introduced for purely computational reasons.

[7] Therefore, if CBDC was introduced into the utility function, the result would be a corner solution with zero CBDC.

The nominal budget constraint is given by

$$\begin{aligned} & \left(B_t^{fi}(j)+D_t^{fi}(j)\right)\left(1+\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)\right) \\ = \; & i_{t-1}B_{t-1}^{fi}(j)+i_{d,t-1}D_{t-1}^{fi}(j)+P_t\Psi_t^u(j) \\ & -P_t c_t^{fi}(j)(1+\tau_{c,t})+W_t^{wo}h_t^{fi}(j)(1-\tau_{L,t})+\frac{\iota}{\varpi}P_t\Upsilon_t(j)\ . \end{aligned} \tag{14}$$

The optimality conditions for aggregate consumption, consumption of domestic and foreign goods, and hours worked are standard, and are omitted to conserve space. The first-order condition for government bonds is given, in real aggregate normalized terms, by

$$\check{\lambda}_t^{fi}\left(1+\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)\right)=\mathbb{E}_t\frac{\beta_{fi}}{x}\left(\check{\lambda}_{t+1}^{fi}r_{t+1}\right)\ , \tag{15}$$

where r is the real policy interest rate, while the condition for wholesale deposits is

$$\check{\lambda}_t^{fi}\left(1+\phi_f\left(b_t^{rat}-\bar{b}_{ss}^{rat}\right)\right)-\gamma\left(\check{d}_t^{fi}\right)^{-\frac{1}{\vartheta}}=\mathbb{E}_t\frac{\beta_{fi}}{x}\left(\check{\lambda}_{t+1}^{fi}r_{d,t+1}\right)\ , \tag{16}$$

where r_d is the wholesale deposit rate, which is higher than the retail rates faced by households. These two conditions imply a spread between the two interest rates due to the monetary convenience yield of financial investors. The level of this spread is assumed to be small but positive, while the interest semi-elasticity of the demand for wholesale deposits is very high.

3.5. Banking Sector

The three functions of the banking sector are wholesale lending and deposit issuance, retail lending, and retail deposit issuance. For analytical convenience, we split banks' optimization problem into these three components, and assign them to different sectors within the banking system. In their interactions with households, domestic banks exclusively extend loans and create deposits in domestic currency. In terms of notation, we need to distinguish per capita stock variables of households and financial investors, which are shown in normal font, from aggregate stock variables, which are shown in bold font. All interbank variables are already in aggregate terms and are therefore shown in normal font.

3.5.1. Wholesale Banks

The function of wholesale banks is to choose the overall size of the banking sector balance sheet to maximize net worth subject to regulatory constraints. They issue wholesale loans in domestic currency to two domestic retail lending banking sectors that in turn lend to domestic and foreign households. Aggregate wholesale household loans are denoted by $\boldsymbol{L}_t^{\ell}(j)=\boldsymbol{L}_{H,t}^{h}(j)+\boldsymbol{L}_{H,t}^{f}(j)$. Wholesale banks also issue interbank loans in domestic currency $L_{H,t}^{b}(j)$ to foreign wholesale banks, and hold interbank deposits in foreign currency $D_{F,t}^{b}(j)$ at foreign retail deposit banks. The gross return to these assets is subject to a lognormally distributed idiosyncratic asset book return shock ω_t^b. The principal liability of wholesale banks consists of wholesale deposits in domestic currency $\boldsymbol{D}_t(j)=\boldsymbol{D}_t^{fi}(j)+\boldsymbol{D}_{H,t}^{h}(j)+\boldsymbol{D}_{H,t}^{f}(j)+D_{H,t}^{b}(j)$ issued to both financial investors and domestic retail deposit banks, where the latter in turn issue deposits to domestic and foreign households and to foreign wholesale banks. The remainder of wholesale banks' liabilities is interbank loans from foreign wholesale banks in foreign currency $L^b_{F,t}(j)$. Their net worth, which is held by domestic households, is $N^b_t(j)$. An individual wholesale bank's balance sheet is therefore given by

$$\boldsymbol{L}^{\ell}_t(j) + L^b_{H,t}(j) + E_t D^b_{F,t}(j) = \boldsymbol{D}_t(j) + E_t L^b_{F,t}(j) + N^b_t(j)\ . \tag{17}$$

Minimum capital adeqacy regulation (MCAR) limits wholesale banks' ability to create credit and money. Bank j faces a future penalty of $\chi\frac{P_{t+1}}{P_t}\left[\boldsymbol{L}^{\ell}_t(j) + L^b_{H,t}(j) + E_t D^b_{F,t}(j)\right]$ if net worth in the next period falls short of Θ times risk-weighted assets in the next period, where the regulatory risk weight on loans to households equals one while the regulatory risk-weight on interbank positions equals $\zeta < 1$. The expression for the share of banks that does not meet MCAR ex-post is not shown to conserve space.

Monetary foreign exchange related transaction costs (MONFX) reflect the fact that banks' exposures to foreign households are costlier to maintain than exposures to domestic households. The reason is that the absence of a lender of last resort in foreign currency requires that banks self-insure, by maintaining readily accessible foreign currency funds in nostro correspondent accounts at foreign banks, to facilitate conversions between foreign and domestic currencies when domestic currency loans to foreigners are made or repaid. This is modelled as a monetary transactions cost that is increasing in loans to foreign households $\boldsymbol{L}^f_{H,t}(j)$ and decreasing in interbank foreign currency liquidity $D^b_{F,t}(j)$. We choose the functional form $s^b_t(j)\boldsymbol{L}^f_{H,t}(j)$, where

$$s^b_t(j) = \frac{\varphi_b}{\vartheta_b}\left(e_t \check{d}^b_{F,t}(j)\right)^{-\vartheta_b}\ . \tag{18}$$

Foreign currency mismatch rules (FXMR) describe banks' management of exposures to foreign residents. As discussed in Kumhof et al. (2020), different kinds of restrictions are conceivable, and they have first-order implications for the behaviour of the model. Here we limit ourselves to a *strict* FXMR rule, which requires interbank positions in foreign currency to be matched at all times, for both countries:

$$D^b_{F,t}(j) - L^b_{F,t}(j) = 0\ . \tag{19}$$

Net worth maximization involves taking first-order conditions with respect to all four asset side items, taking interest rates as given. Gross nominal wholesale lending rates are denoted by $i^x_{\ell X,t}$, and can be interpreted as the rates banks would charge to nominally riskless borrowers (which are not present in the model). The gross wholesale money market interest rate facing banks is denoted by $i_{w,t}$, and the terms $\Pi^R_t(j)$ and $\Lambda^b_t(j)$ denote the net ex-post profits of retail deposit banks and the net ex-post losses of retail lending banks. Banks internalize the risk of breaching the MCAR, so that expected net worth includes the penalty payable if a breach occurs, weighted by the probability of a breach. We have the following optimization problem:

$$\max_{\left\{\begin{array}{c}\boldsymbol{L}^h_{H,t}(j),\ \boldsymbol{L}^f_{H,t}(j),\\ L^b_{H,t}(j),\ D^b_{F,t}(j)\end{array}\right\}} \mathbb{E}_t\left[\begin{array}{c}\left[i^h_{\ell H,t}\boldsymbol{L}^h_{H,t}(j) + i^f_{\ell H,t}\boldsymbol{L}^f_{H,t}(j) + i^b_{\ell H,t}L^b_{H,t}(j) + E_{t+1}i^b_{dF,t}D^b_{F,t}(j)\right]\omega^b_{t+1}\\ -i_{w,t}\boldsymbol{D}_t(j) - E_{t+1}i^b_{\ell F,t}L^b_{F,t}(j)\\ -s^b_t(j)\boldsymbol{L}^f_{H,t}(j) + P_{t+1}\left(\Pi^R_{t+1}(j) - \Lambda^b_{t+1}(j)\right)\\ -P_{t+1}\int_0^{\bar{\omega}^b_{t+1}(j)}\frac{\chi}{P_t}\left(\boldsymbol{L}^h_{H,t}(j) + \boldsymbol{L}^f_{H,t}(j) + L^b_{H,t}(j) + E_t D^b_{F,t}(j)\right)f^b\left(\omega^b_{t+1}\right)d\omega^b_{t+1}\end{array}\right] \tag{20}$$

The deposit terms in this expression must be replaced using a combination of the balance sheet identity (17) and the FXMR rule (19). Post-dividend net worth equals the above expression minus dividends that equal a fixed fraction of net worth, and that are paid out to households in a lump-sum fashion, a specification that can be obtained by applying the "extended family" approach of Gertler and Karadi (2011). The law of motion for wholesale bank net worth is not shown to conserve space.

Optimization yields first-order conditions that we show in full, because they reveal important details concerning the structure of interest rate spreads. We can drop individual indices because in equilibrium the ratios to net worth of loans, deposits, retail deposit profits and retail lending losses are identical across banks. We therefore show the conditions in real normalized form. The expressions $\check{\Omega}^x_{yX,t}$ are the derivatives $\partial\bar{\omega}^b_{t+1}/\partial\check{y}^x_{X,t}$, with $y \in \{\ell, d\}$. We note that $\check{\Omega}^x_{yX,t}$ are always positive, that they are very similar in size between the two types of wholesale loans and separately between the two types of interbank positions, and finally that they are smaller for interbank positions than for wholesale loans, due to the lower regulatory risk weight on interbank positions.

For domestic currency loans to domestic households $\check{\boldsymbol{\ell}}^h_{H,t}$ we have

$$\mathbb{E}_t \left\{ r^h_{\ell H,t+1} - r_{w,t+1} - \chi \left[F^b_{t+1} + f^b_{t+1} \check{\Omega}^h_{\ell H,t} \left(\check{\boldsymbol{\ell}}^\ell_t + \check{\ell}^b_{H,t} + e_t \check{d}^b_{F,t} \right) \right] \right\} = 0 \; . \tag{21}$$

This condition contains a *regulatory spread* $-\chi \left[F^b_{t+1} + f^b_{t+1} \check{\Omega}^h_{\ell H,t} \left(\check{\boldsymbol{\ell}}^\ell_t + \check{\ell}^b_{H,t} + e_t \check{d}^b_{F,t} \right) \right]$ between the wholesale lending rate and the policy rate, whereby the wholesale lending rate must compensate wholesale banks for the fact that that at the margin an additional loan increases the penalty payable in case of a breach of MCAR. The size of this spread can be shown to depend on a combination of the size of the MCAR (γ), the penalty payable in case of a breach of the MCAR (χ), and the likelihood of a breach given the riskiness of individual banks (F^b_{t+1} and f^b_{t+1}). This condition is identical across all versions of FXMR.

For domestic currency loans to foreign households $\check{\boldsymbol{\ell}}^f_{H,t}$, we have

$$\mathbb{E}_t \left\{ r^f_{\ell H,t+1} - r_{w,t+1} - s^b_t/\pi^p_{t+1} - \chi \left[F^b_{t+1} + f^b_{t+1} \check{\Omega}^f_{\ell H,t} \left(\check{\boldsymbol{\ell}}^\ell_t + \check{\ell}^b_{H,t} + e_t \check{d}^b_{F,t} \right) \right] \right\} = 0 \; . \tag{22}$$

This condition contains a regulatory spread that is virtually identical in size to that for loans to domestic households. But in addition there is an *interbank monetary spread* $\left(s^b_t/\pi^p_{t+1}\right)$, which arises because an increase in exposures to foreign households must be matched with a costly increase in foreign currency interbank liquidity.

For domestic currency loans to foreign banks $\check{\ell}^b_{H,t}$ we have

$$\mathbb{E}_t \left\{ r^b_{\ell H,t+1} - r_{w,t+1} - \chi \left[F^b_{t+1} + f^b_{t+1} \check{\Omega}^b_{\ell H,t} \left(\check{\boldsymbol{\ell}}^\ell_t + \check{\ell}^b_{H,t} + e_t \check{d}^b_{F,t} \right) \right] \right\} = 0 \; . \tag{23}$$

In this case the regulatory spread is significantly smaller, due to a much lower Basel risk weight on interbank loans. For foreign banks, the interest rate $r^b_{\ell H,t+1}$ partly determines the marginal cost of refinancing domestic currency interbank deposits at domestic banks - see (24) below.

For foreign currency deposits at foreign banks $\check{d}^b_{F,t}$ we have

$$\mathbb{E}_t \left\{ r^b_{dF,t+1} - r^b_{\ell F,t+1} - s^{b\prime}_t \check{\boldsymbol{\ell}}^f_{H,t}/\pi^p_{t+1} - \chi \left[F^b_{t+1} + f^b_{t+1} \check{\Omega}^b_{dF,t} \left(\check{\boldsymbol{\ell}}^\ell_t + \check{\ell}^b_{H,t} + e_t \check{d}^b_{F,t} \right) \right] \right\} = 0 \; . \tag{24}$$

Because of FXMR, foreign currency interbank loans at the rate $i^b_{\ell F,t}$, rather than domestic wholesale deposits at the rate $i_{w,t}$, are the marginal source of refinancing foreign currency interbank deposits. The regulatory spread is virtually identical in size to that of domestic currency interbank loans. In equilibrium this spread is however more than offset by the *interbank monetary discount* $s^{b\prime}_t(\check{\boldsymbol{\ell}}^f_{H,t}/\pi^p_{t+1}) < 0$. The reason for this discount is that holdings of foreign currency interbank deposits reduce the cost of exposures to foreign households.

3.5.2. Retail Deposit Banks

The function of retail deposit banks is to set the terms of retail deposit contracts. Retail deposit banks purchase domestic currency wholesale deposits, and pay for them by issuing retail and interbank deposit varieties in domestic currency, which are denoted by $D^h_{H,t}(j)$, $D^f_{H,t}(j)$ and $D^b_{H,t}(j)$. Domestic currency wholesale deposits are placed through the domestic wholesale money market, at the deposit rate determined by the preferences for net wholesale deposits of perfectly competitive financial investors $i_{d,t}$. Financial investors' stock of gross wholesale deposits, because it includes wholesale deposits of retail deposit banks that are placed through the wholesale money market, is larger than their stock of net wholesale deposits. The following arbitrage condition holds for wholesale money market rates:

$$i_{w,t} = i_{d,t} \ . \tag{25}$$

Retail deposit banks behave as monopolistic competitors towards the holders of their retail and interbank deposits, who demand CES composites of all deposit varieties. This implies the pricing rules for deposits

$$\begin{aligned} i^h_{dH,t} &= \mu^h_{dH} i_{w,t} \ , \\ i^f_{dH,t} &= \mu^f_{dH} i_{w,t} \ , \\ i^b_{dH,t} &= \mu^b_{dH} i_{w,t} \ , \end{aligned} \tag{26}$$

where the markdown terms μ^x_{dH} are smaller than one. Retail deposit banks are fully owned by wholesale banks, and their aggregate real profits $\check{\Pi}^R_t$, which are positive in steady state due to retail deposit banks' market power, are transferred lump-sum to the latter.

3.5.3. Retail Lending Banks

The function of retail lending banks is to set the terms of loan contracts, in a formally very similar fashion to Bernanke et al. (1999), henceforth BGG, whose set-up we follow and extend. There are two retail lending bank sectors, one each for loans to domestic and foreign households. These two groups of borrowers each have unit mass and are indexed by j, where individual borrowers may differ by the size of their internal funds and therefore their borrowing capacity. Retail lending banks themselves are homogenous, and each bank lends the same amount to borrower j.

The total potential collateral for a loan contract between retail lending banks and borrower j is the value of capital $Q_t k_t(j)$ held by that borrower, where Q_t is the nominal price of capital and $k_t(j)$ is the stock of capital owned by j. While in most frameworks that use the BGG model the available collateral is assumed to consist of 100% of the total market value of collateral at all times, in our framework that fraction does not necessarily equal 100% in steady state, is subject to stochastic shocks, and is split into two parts, one to collateralize domestic currency loans from domestic banks and another to collateralize foreign currency loans from foreign banks. Specifically, domestic and foreign households j are able to pledge time-varying shares $\kappa_{H,t}^{h}$ and $\kappa_{H,t}^{f*}$ (which do not necessarily sum to 100%) of land to domestic banks in order to collateralize loans $L_{H,t}^{h}(j)$ and $L_{H,t}^{f}(j)$.

Domestic retail lending banks' loan contracts stipulate non-contingent retail lending rates $i_{rH,t}^{x}$ on loans $L_{H,t}^{x}(j)$ that must be paid in full if the realization of the shock is sufficiently high to avoid bankruptcy. Borrowers decide to declare bankruptcy if their individual productivity shock remains below $\bar{\omega}_{H,t}^{x}$. In that case handing over the entire value of their project over to the bank becomes preferable to realizing the project and repaying the loan. We omit the closed form expression for $\bar{\omega}_{H,t}^{x}$ to conserve space. In case of bankruptcy, because of monitoring costs, the bank can only recover a fraction $1-\xi_{H}^{x}$ of the project value, where ξ_{H}^{x} is the loss-given-default percentage. The cost of funding for retail lending banks is given by wholesale lending rates $i_{\ell H,t}^{x}$. The participation constraints for retail loans state that wholesale returns must equal the sum of gross interest on fully repaid loans weighted by the probability of of full repayment, plus the value of pledged collateral net of monitoring costs recoverable in case of default. After some algebra, these constraints can be rewritten in the form of (8) above.

The participation constraints state that ex-ante net loan losses must equal zero. Ex-post net loan losses can be different from zero because lending rates are non-contingent. Retail lending banks are fully owned by wholesale banks, and their net aggregate loan losses $\check{\Lambda}_{t}^{b}$ are transferred to the latter in a lump-sum fashion. The participation constraints enter into households' optimization problems, and affect their decision rules for capital and loans.

3.5.4. Cross-Border Financial Markets

Domestic and foreign households and banks are linked through balance sheet positions and in terms of interest rates. For Home, the nominal interest rates on foreign currency loans and deposits are identical to those prevailing in Foreign, $i_{\ell F,t}^{x}=i_{\ell F,t}^{x*}$ and $i_{dF,t}^{x}=i_{dF,t}^{x*}$, where $x\in\{h,b\}$. The corresponding cross-border balance sheet positions are $\check{\ell}_{F,t}^{x}=\check{\ell}_{F,t}^{x*}\frac{1-n}{n}$ and $\check{d}_{F,t}^{x}=\check{d}_{F,t}^{x*}\frac{1-n}{n}$. Analogous relationships hold for Home currency exposures in Foreign, as well as for CBDC exposures and CBDC interest rates.

3.6. Manufacturers and Unions

Manufacturers have unit mass and are indexed by j, where individual manufacturers differ by the goods variety that they produce and sell. Manufacturers optimally combine labour $h_t(j)$ and capital $K_t(j)$ to produce and price varieties of the domestic good $y_t(j)$. They set the price of their output variety $P_t(j)$ subject to monopolistic competition and Ireland (2002) quadratic inflation adjustment costs, with buyers demanding a composite of output varieties with elasticity of substitution θ_p. The production function for manufacturer j is given by

$$y_t(j)=\left(S_t^{a}T_t h_t(j)\right)^{1-\alpha}\left(K_t(j)\right)^{\alpha}\ , \tag{27}$$

where S_t^a is a first-order autoregressive stochastic process for labour-augmenting technology. Shocks to the latter are the main aggregate supply shock in the model. The technology (27) implies standard conditions for real marginal cost and for factor demands. We assume that manufacturers adopt local currency pricing in both domestic and foreign markets. Maximization of discounted nominal profits with respect to $P_{H,t}(j)$ and $P^*_{H,t}(j)$ yields a pair of standard New Keynesian Phillips curves. Manufacturers are fully owned by households, and their profits $\check{\Pi}_t^M$ are transferred to the latter in a lump-sum fashion.

Unions have unit mass and are indexed by j, where individual unions differ by the labour variety they sell. Unions buy homogenous labor from households at a nominal worker wage rate W_t^{wo} set in a competitive labor market. They set the price of their labor variety $W_t^{pr}(j)$ subject to monopolistic competition and quadratic wage inflation adjustment costs, with manufacturers demanding a composite of labor varieties with elasticity of substitution θ_w. Maximization of discounted nominal profits with respect to $W_t^{pr}(j)$ yields a standard wage Phillips curve. Unions are owned by households and financial investors, and their profits $\check{\Pi}_t^U$ are transferred to these two groups in proportion to their contribution to the labor supply, and in a lump-sum fashion.

3.7. Monetary and Fiscal Policy

3.7.1. Monetary Policy

The central bank separately determines the traditional interest rate on reserves and either the interest rate on or the quantity of CBDC.

Policy Rule for the Interest Rate on Reserves: Under CBDC the main policy tool remains the interest rate on central bank reserves. It is assumed to follow a conventional inflation forecast-based interest rate rule, with steady state nominal interest rate $\bar{\imath}$ (derived from financial investors' first-order condition for government bonds), interest rate smoothing, a countercyclical response to deviations of three-quarters-ahead inflation from the inflation target $\bar{\pi}$, and a first-order autoregressive monetary policy shock S_t^{int}:

$$i_t = (i_{t-1})^{i_i} \, \bar{\imath}^{(1-i_i)} \left(\frac{\left(\pi_{t+3}\pi_{t+2}\pi_{t+1}\pi_t\right)^{1/4}}{\bar{\pi}} \right)^{(1-i_i)i_\pi} S_t^{int} \; . \tag{28}$$

Policy Rules for CBDC: The debate about whether to use the interest rate on reserves or a monetary aggregate as a monetary policy tool was settled in favor of the former decades ago. This was based on three arguments. First, there would be problems in defining a monetary aggregate whose control would represent an economically relevant lever. However, this need not apply to CBDC as long as the quantity outstanding is sufficiently large and its substitutability with other monetary transactions media is sufficiently low to make it a relevant lever. Second, even if the relevant monetary aggregate could be defined, there would be problems in controlling it effectively, given that all but the narrowest monetary aggregates are under the control of private banks rather than the central bank. This includes the possibility, known as Goodhart's Law (Goodhart (1975)), that private sector behavior may change in response to changes in the targeted monetary aggregate. This is closely related to the Lucas critique (Lucas (1976)). There is of course no claim that controlling CBDC amounts to controlling broader monetary aggregates. But as long as the control of CBDC represents an economically relevant lever, this is not a problem because its quantity can be directly controlled by the issuing central bank. Third, Poole (1970) argued that controlling a quantity aggregate must lead to more aggregate volatility than controlling an interest rate if shocks to money demand are sufficiently important. As we will show, this argument does apply to CBDC, but in a far more attenuated form than in Poole (1970). This is because under a CBDC regime banks remain the creators of the marginal unit of money, while in the framework of Poole (1970) broad money was assumed to be controlled by government.

In order to study the third question in more detail, we need to specify CBDC policy rules. In interpreting these rules, a key insight is that a higher interest rate on CBDC triggers an increased supply of CBDC and is therefore stimulative rather than contractionary. This is because with an additional supply of CBDC, liquidity becomes less scarce, so that the convenience yield of CBDC drops and therefore, for a given opportunity cost, its financial yield $i_{m,t}$ must increase.

Under a CBDC quantity rule, the central bank fixes the ratio of CBDC to GDP at a target value of $\overline{m^{rat}}$ over the cycle, and it may in addition permit it to vary countercyclically. The rule is

$$m_t^{rat} = \bar{m}^{rat} - 400 m_\pi E_t \ln \left(\frac{\left(\pi_{t+3}\pi_{t+2}\pi_{t+1}\pi_t\right)^{1/4}}{\bar{\pi}} \right) , \tag{29}$$

where $m_t^{rat} = 100 \left(m_t / \left(4gdp_t \right) \right)$ and $m_\pi \geq 0$. The baseline version of this rule, with $m_\pi = 0$, implies a fixed quantity of CBDC relative to GDP, so that any changes in demand for CBDC will be reflected in the interest rate on CBDC $i_{m,t}$ alone, except to the extent that they affect GDP. For $m_\pi > 0$, when inflation is expected to be above target, this rule removes CBDC from circulation, through a central bank sale to the private sector of government debt against CBDC. This draining of purchasing power has a countercyclical effect that goes beyond the effects of the policy rate i_t, because it increases the effective price of consumption.

Under a CBDC interest rate rule, the central bank varies the nominal interest rate paid on digital currency according to$i_{m,t} = \frac{i_t}{\mathfrak{sp}} \left(\frac{\pi_{4,t+3}}{\bar{\pi}} \right)^{-m_\pi}$

$$i_{m,t} = \frac{i_t}{\mathfrak{sp}} \left(\frac{\left(\pi_{t+3}\pi_{t+2}\pi_{t+1}\pi_t\right)^{1/4}}{\bar{\pi}} \right)^{-i_\pi^m} . \tag{30}$$

The baseline version of this rule, with $i_\pi^m = 0$, implies a fixed spread $\mathfrak{sp} > 1$ of the policy rate relative to the CBDC interest rate, so that any changes in demand for CBDC will be reflected in the quantity of CBDC m_t alone, except to the extent that they affect the policy rate. For $i_\pi^m > 0$, when inflation is expected to be above target, the interest rate on CBDC is *lowered* relative to the policy rate. This, ceteris paribus, makes CBDC less attractive, so that agents will exchange it for government bonds. This endogenous reduction in liquidity has the same effects as the direct withdrawal of liquidity under the countercyclical quantity rule.

Both rules are currently set in order to stabilize inflation, just like the rule for the interest rate on reserves, and we will show that such rules can indeed make a significant contribution to the stabilization of inflation. However, issuance of CBDC provides the central bank with a second instrument that could potentially be employed to achieve other objectives, such as financial stability targets.

Issuance Arrangements for CBDC: A key practical concern among policymakers has been the perceived risk of a system-wide run from bank deposits to CBDC (runs on individual institutions are of course possible with or without CBDC). A frequent partial equilibrium fallacy is the argument that holders of bank deposits can, for technological reasons, run into CBDC much more quickly than into cash, thereby increasing systemic risk. This does not survive general equilibrium analysis if the only available counterparties are other private-sector agents, in which case the "run" is merely a reallocation of unchanged stocks of deposits and CBDC among different agents. For a system-wide run, it is therefore necessary that the central bank itself adopts *issuance arrangements* whereby it accepts bank deposits in payment for CBDC, and that it adopts a *policy rule* that elastically accommodates large-scale changes in CBDC demand, thereby potentially becoming a system-wide and ultimately unsecured lender of last resort. Central banks have never issued central bank money under such arrangements, and Kumhof and Noone (2021) argue that they should not start doing so under CBDC. Instead, that paper advocates core principles that minimize and largely eliminate the risk of system-wide runs. The first line of defense is the policy rule, which should feature an adjustable CBDC interest rate that allows the market for CBDC to clear without a need for either large balance sheet adjustments or large movements in the general price level. A quantity rule could completely eliminate runs into CBDC through lower CBDC interest rates, as long as the necessary interest rate can remain within acceptable bounds. And even under an interest rate rule, rate setting could help dampen large fluctuations in CBDC demand. If this is nevertheless insufficient, the second line of defense is the issuance arrangements, which should only guarantee central bank issuance of CBDC against eligible securities, principally government securities, whereas it should not guarantee on-demand convertibility of bank deposits into CBDC.[8] Households and firms would be able to freely trade bank deposits against CBDC in a private market, and that private market could freely obtain additional CBDC from the central bank, at the posted CBDC interest rate and against eligible securities. During normal times the central bank could also trade CBDC against bank deposits in this market, but at its discretion. The withdrawal of the central bank from that market during times of stress would be the equivalent of a bank holiday during a cash-driven run in a traditional banking system.[9] Our theoretical model embeds the issuance arrangement assumptions of Kumhof and Noone (2021). Specifically, the government budget constraint reflects that the government does not issue CBDC against bank deposits, the monetary policy rule for the interest rate on reserves implies that the market for central bank reserves is separate from the market for CBDC, and both alternatives for CBDC imply that the CBDC interest rate is adjustable.

3.7.2. Fiscal Policy

The consolidated public sector budget constraint is, in real normalized form, given by

$$\check{b}_t + \check{m}_t = \frac{r_t}{x}\check{b}_{t-1} + \frac{r_{m,t}}{x}\check{m}_{t-1} + p_{H,t}\check{g}_t + trf_t - \check{\tau}_t \ . \tag{31}$$

Government bonds and CBDC enter in identical fashion, but with the important difference that the interest rate on CBDC is considerably lower than the interest rate on government bonds, due to the convenience yield of CBDC. This budget constraint shows explicitly that CBDC can only be issued to the public in two ways, through flow transfers from the central bank to the government that help to finance primary deficits, or through stock purchases of government bonds by the central bank. Bank deposits do not enter, so that a system-wide run from bank deposits to CBDC becomes impossible.

[8] It can be shown that guaranteed on-demand convertibility of reserves into CBDC would also need to be ruled out, as this could still facilitate system-wide bank runs.

[9] Kumhof and Noone (2021) also discuss the alternative arrangements proposed by Bindseil (2020).

Tax revenues are given by

$$\check{\tau}_t = \tau_t^{ls} + \tau_{c,t}\check{c}_t + \tau_{L,t}\check{w}_t^{wo}h_t + \tau_{k,t}\left(r_{k,t} - \Delta q_t\right)\frac{\check{\boldsymbol{k}}_{t-1}}{x} \,, \tag{32}$$

which shows clearly the potential benefits of applying lower government financing costs towards reductions in distortionary taxes. CBDC issuance against government debt can accomplish lower financing costs in two ways. First, by increasing the share of financing that pays the lower interest rate on CBDC. And second, by reducing the outstanding stock of defaultable government debt and thereby reducing all equilibrium interest rates. As discussed in the introduction, and as argued by Kumhof et al. (2021), government debt is defaultable and is a liability of government, while CBDC is not defaultable and is not a liability of government but rather a hybrid instrument that is closer to equity (in the nation) rather than debt.

A fiscal rule enforces stability of the deficit-to-GDP ratio

$$gd_t^{rat} = \overline{gd^{rat}} - 100d^{gdp}\ln\left(\frac{g\check{d}p_t}{gdp_{ss}}\right) \,, \tag{33}$$

where allowance is made for automatic stabilizers induced by the output gap term $ln\left(g\check{d}p_t/gdp_{ss}\right)$, where $gd_t^{rat} = gd_{b,t}^{rat} + gd_{m,t}^{rat}$, and where $gd_{b,t}^{rat} = 100(B_t - B_{t-1})/GDP_t$ and $gd_{m,t}^{rat} = 100(M_t - M_{t-1})/GDP_t$. Treating changes in CBDC in identical fashion to changes in government debt turns out to be very important to fiscal stability, because it prevents large inflows or outflows of CBDC seigniorage from destabilizing fiscal instruments. For our shock simulations we will abstract from fiscal considerations, by using (33) to endogenize lump-sum taxes while keeping all distortionary tax rates constant. But for our transition simulation fiscal issues are central, and we will use (33) to endogenize $\tau_{L,t}$, with auxiliary rules used to move the other distortionary tax rates in proportional fashion

$$\frac{\tau_{c,t} - \bar{\tau}_c}{\bar{\tau}_c} = \frac{\tau_{L,t} - \bar{\tau}_L}{\bar{\tau}_L} \quad , \quad \frac{\tau_{k,t} - \bar{\tau}_k}{\bar{\tau}_k} = \frac{\tau_{L,t} - \bar{\tau}_L}{\bar{\tau}_L} \,, \tag{34}$$

and with government spending equal to a constant share s_g of steady state GDP.

3.8. Market Clearing including Balance of Payments

The market clearing condition for aggregate Home goods output is $\check{y}_t = \check{y}_{H,t} + \check{y}_{H,t}^*$. The market clearing condition for Home goods sold in Home takes account of the fact that a share $\mathfrak{r}$ of monetary and interbank transaction costs, regulatory penalties due to MCAR breaches, and bankrupcy monitoring costs, with their sum denoted by $\check{\mathfrak{I}}_t^{res}$, represent real resource costs rather than lump-sum rebates, so that we have $\check{y}_{H,t} = \check{c}_{H,t} + (1-\varpi)\check{I}_t + \check{g}_t + \mathfrak{r}\check{\mathfrak{I}}_t^{res}/p_{H,t}$. For Home goods sold in Foreign, we instead have $\check{y}_{H,t}^* = \check{c}_{H,t}^*\frac{1-n}{n}$. For both Home and Foreign consumption goods, we have $\check{c}_{X,t} = \varpi\check{c}_{X,t}^{fi} + (1-\varpi)\check{c}_{X,t}^{hh}$. We also have market clearing conditions for hours, $h_t = \varpi h_t^{fi} + (1-\varpi)h_t^{hh}$, capital $\check{\boldsymbol{k}}_t = (1-\varpi)\check{k}_t$, bonds $\check{b}_t = \varpi\check{b}_t^{fi}$, and CBDC $\check{m}_t = \check{\boldsymbol{m}}_{H,t}^h + \check{\boldsymbol{m}}_{H,t}^f$. Nominal GDP deflated by the CPI price index is given by

$$g\check{d}p_t = \check{c}_t + (1-\varpi)p_{H,t}\check{I}_t + p_{H,t}\check{g}_t + e\check{x}p_t - i\check{m}p_t \tag{35}$$

where real exports are $\check{x}_t = e_t p_{H,t}^* c_{H,t}^* \frac{1-n}{n}$ and real imports are $\check{m}_t = p_{F,t}\check{c}_{F,t}$. In our simulations the concept of GDP in (35) is only used to compute ratios to GDP of other CPI-deflated variables, such as balance sheet ratios. The variable "real GDP" in our charts is based on a Fisher index. The current account equation, in real normalized terms, can be shown to be given by

$$\begin{aligned}
&\left(\check{\boldsymbol{\ell}}^{f}_{H,t}+\check{\ell}^{b}_{H,t}\right)+e_t\left(\check{\boldsymbol{m}}^{h}_{F,t}+\check{\boldsymbol{d}}^{h}_{F,t}+\check{d}^{b}_{F,t}\right)-\left(\check{\boldsymbol{m}}^{f}_{H,t}+\check{\boldsymbol{d}}^{f}_{H,t}+\check{d}^{b}_{H,t}\right)-e_t\left(\check{\boldsymbol{\ell}}^{h}_{F,t}+\check{\ell}^{b}_{F,t}\right) \\
&=\frac{1}{x}\left(r^{f}_{\ell H,t}\check{\boldsymbol{\ell}}^{f}_{H,t-1}+r^{b}_{\ell H,t}\check{\ell}^{b}_{H,t-1}+r^{h}_{mF,t}e_{t-1}\check{\boldsymbol{m}}^{h}_{F,t-1}+r^{h}_{dF,t}e_{t-1}\check{\boldsymbol{d}}^{h}_{F,t-1}+r^{b}_{dF,t}e_{t-1}\check{d}^{b}_{F,t-1}\right) \\
&-\frac{1}{x}\left(r^{f}_{mH,t}\check{\boldsymbol{m}}^{f}_{H,t-1}+r^{f}_{dH,t}\check{\boldsymbol{d}}^{f}_{H,t-1}+r^{b}_{dH,t}\check{d}^{b}_{H,t-1}+r^{h}_{\ell F,t}e_{t-1}\check{\boldsymbol{\ell}}^{h}_{F,t-1}+r^{b}_{\ell F,t}e_{t-1}\check{\ell}^{b}_{F,t-1}\right) \\
&+\check{\Lambda}^{h}_{t}-\check{\Lambda}^{b}_{t}+e_t p^{*}_{H,t}\check{c}^{*}_{H,t}\frac{1-n}{n}-p_{F,t}\check{c}_{F,t}
\end{aligned} \tag{36}$$

where the first line is the net foreign asset position, which consists of ten gross positions. The change in the net foreign asset position equals the sum of net interest payments on the ten gross positions, plus (small) cross-border components of ex-post loan loss rebates to households minus loan losses of banks, plus the trade balance.

3.9. Key Elasticities

Two elasticities are critical for the calibration of liquidity demand in the model, the interest semi-elasticity of financial investors' demand for wholesale deposits and the elasticity of household consumption with respect to total liquidity. We will deal with each of these in turn.

3.9.1. Interest Semi-Elasticity of the Demand for Wholesale Deposits

This elasticity measures the increase, in percent, of financial investors' demand for wholesale deposits in response to a one percentage point decrease in the annualized opportunity cost of deposits, specifically in the spread between the policy rate and the wholesale deposit rate. We combine the steady state versions of financial investors' Euler equations for bonds and deposits, take logs, evaluate the derivative $d\ln\left(d^{fi}\right)/d\ln\left(r_d/r\right)$, and take account of the fact that model interest rates are in quarterly rather than annual terms, to obtain the following expression for this elasticity:

$$\epsilon^{fi}_{d}=\vartheta\left(\frac{\left(d^{fi}\right)^{\frac{1}{\vartheta}}\lambda^{fi}}{\gamma}-1\right)/4\ . \tag{37}$$

3.9.2. Elasticity of Household Consumption with Respect to Total Liquidity

This elasticity measures the increase, in percent, of households' consumption demand in response to a one percent increase in the quantity of total liquidity. We use the household sector's consumption optimality condition in steady state, hold the marginal utility of wealth constant and evaluate the derivative $d\ln\left(c^{hh}\right)/d\ln\left(liq\right)$. We obtain the following expression:

$$\epsilon^{hh}_{c}=\frac{s^{c}\mathfrak{m}(1+\mathfrak{m})}{1+s^{c}\mathfrak{m}(1+\mathfrak{m})}\ . \tag{38}$$

4. Calibration

The baseline calibration of the pre-CBDC economy is for a symmetric 2-country model and closely matches the one in Kumhof et al. (2020), who use a combination of US and international data, together with established parameter values from the literature, to calibrate the model.

4.1. Real Sector Parameters

World trend productivity growth x is calibrated at 2% in annual terms, while the (pre-CBDC) equilibrium real interest rate $\bar{r}$ and the CPI inflation target $\bar{\pi}$ are calibrated at 3% and 2% in annualized terms, respectively, the latter directly and the former by adjusting the discount factor of financial investors β_{fi}.

For preferences, we normalize the steady state labour supplies of both households and financial investors to 1 by adjusting the preference weights ψ_{hh} and ψ_{fi}, and we equalize steady state per capita consumption of households and financial investors by adjusting the lump-sum transfer parameter ι. We set the steady state share of foreign goods in the domestic consumption basket to 15.3% by adjusting the goods market home bias parameter b^c. The 15.3% value matches the average ratio of imports to total final expenditure in the US over the period 2000-2016. The elasticity of substitution θ_c between domestic and foreign goods is set to 1.5, a common value in the open economy macro literature. We set the elasticity of labour supply η to 1. The habit parameter ν is calibrated at 0.74, based on the estimation results of Christiano et al. (2014).

For technologies, we set the steady state capital income share to 40% by adjusting the share parameter α. We also set the ratio of investment to GDP to 20% by adjusting the depreciation rate Δ. We set both price and wage gross markups, μ_p and μ_w, to 1.1. Together with price and wage inflation stickiness parameters of $\phi_p = 200$ and $\phi_w = 200$, this implies an average duration of price and wage contracts of 5 quarters in an equivalent Calvo (1983) setup with full indexation to past inflation. This is similar to the results of Christiano *et al.* (2005). The investment adjustment cost parameter, at $\phi_I = 2.5$, follows Christiano *et al.* (2005). The share of transactions and adjustment costs that represents real resource cost is set to equal $\mathfrak{r} = 0.25$.

The initial steady state government debt-to-GDP ratio is set to 80% by adjusting $\overline{gd^{rat}}$, roughly equal to its value prior to the onset of the Great Recession. Laubach (2009), Engen and Hubbard (2004) and Gale and Orszag (2004) report empirical estimates, for the United States, of the elasticity (in percent p.a.) of the real policy interest rate with respect to changes in the government debt-to-GDP ratio (in percentage points). They report that each percentage point increase in the debt ratio increases the real interest rate by between 1 and 6 basis points. We calibrate this elasticity conservatively at 2 basis points, which requires setting $\phi_f = 0.00005$. For a clearer interpretation of the transition results, we impose a balanced budget fiscal rule by setting $d^{gdp} = 0$. We impose the same for the shock simulations, but here it does not matter because only lump-sum taxes adjust to satisfy the fiscal rule. The steady state share of government spending in real GDP is set to a constant 18% by adjusting s_g. Steady state tax rates on labor, capital and consumption $\tau_{L,t}$, $\tau_{k,t}$ and $\tau_{c,t}$ are adjusted to reproduce the historical ratios of the respective tax revenues to GDP. Given these calibration targets, net lump-sum taxes are adjusted to balance the budget in the initial steady state, and are thereafter held constant relative to steady state GDP. For the monetary policy reaction function we stay close to the coefficient estimates for the Federal Reserve Board's SIGMA model (Erceg *et al.* (2006)) and the IMF's Global Projection Model (Carabenciov *et al.* (2013)), with $i_i = 0.7$ and $i_\pi = 2.0$.

4.2. Financial Sector Parameters

The first subset of financial sector calibration targets relates to prudential regulation and the riskiness of banks and their borrowers. The capital adequacy ratio Θ is set to 10.5% which is the sum of the 8.0% Basel III total capital ratio and the 2.5% capital conservation buffer (see Basel Committee on Banking Supervision (2017)). We omit the countercyclical and GSIB buffers, and the additional supervisory requirements, as these do not apply to all banks and/or at all times. In our setup banks will end up holding considerable capital above the regulatory minimum to self-insure against the risk of violating the MCAR. We therefore set the actual steady state capital adequacy ratio to 15.5%, based on recent data[10], by adjusting banks' dividend payout parameter δ^b. The cumulative share of banks that violate the Basel minimum in any quarter is F^b_{t+1}. We set it to 2.5% in steady state, close to the approximate historical frequency of systemic banking crises in Jorda et al. (2011), by adjusting the bank riskiness parameter σ^b. Bankruptcy rates of domestic and foreign bank borrowers F^x_X are set at 1.5% in steady state by adjusting the loss-given-default parameters ξ^x_X. This matches the findings of Ueda and Brooks (2011) for non-financial listed US companies, and also approximately matches the historical average of per-capita default rates reported in Albanesi et al. (2017).

The second subset of financial sector calibration targets relates to steady-state interest-rate spreads, which are summarized in Figure 2. We set the spread between interbank lending rates and the policy rate to 25 basis points, close to the 16 basis points average spread between the 1-month LIBOR and the effective Federal Funds rate over the period 2000-2016, by adjusting the parameter ζ, the Basel risk weight on interbank claims. The implied risk weight is $\zeta = 0.28$. It was given a value of 20 percent in the Basel I rules, but subsequent versions of the Basel rules made this parameter model-based or risk-sensitive, and hence non-unique (see Basel Committee on Banking Supervision (2017)). We set the spread between the policy rate and the wholesale deposit rate to 25 basis points, by adjusting financial investors' deposits-in-the-utility function parameter γ.

To calibrate the remaining lending spreads, we use a data set, constructed by Anderson and Cesa-Bianchi (2020), of "maturity-adjusted credit spreads" (MACS) for all listed non-financial firms in the US. These are spreads between the cost of borrowing for a given firm and an equal-maturity risk-free interest rate. We recall that in the model the wholesale lending rate corresponds to the interest rate that would be charged to a notional zero-risk corporate borrower. A model-consistent calibration for wholesale lending spreads is therefore the spread between the average commercial paper rate paid by the very best blue-chip (AAA) non-bank corporates and the policy rate at matching maturities of 3 months. Over the sample period 2000-2016 we compute this as 66 basis points, and calibrate the wholesale lending spread charged to domestic households to this value, by adjusting the MCAR parameter χ . The wholesale lending spread charged to foreign households also contains a regulatory spread of 66 basis points, but in this case there is an additional small MONFX spread. We calibrate this at 10 basis points, by adjusting the willingness-to-lend parameter for loans to foreign households κ^f_H. The total foreign household wholesale spread therefore equals 76 basis points.

[10] See Federal Reserve Bank of New York (2018).

External finance premia or retail lending spreads, which are the spreads between the retail and wholesale lending rates, are calibrated by adjusting the borrower riskiness parameters σ_H^h and σ_H^f. Their data counterpart is the difference between the MACS of all firms and the MACS of AAA-rated firms only, again at matching maturities of 3 months. In our data set this difference is equal to 167 basis points[11], which we use to calibrate the spread on domestic household retail loans. This yields a domestic household retail lending rate of 5.33% . The spread on foreign household retail loans is calibrated at 132 basis points, so that the foreign household retail lending rate equals 5.08%. The lower retail spread for foreign relative to domestic household loans is justified by the fact that foreign loans are taken out by larger corporates that are more creditworthy than the average domestic currency borrower.

Turning to liability-side rates, the steady state spread between the domestic policy rate and the domestic currency interbank deposit rate is calibrated to equal the 25 basis point spread between the policy rate and the wholesale deposit rate, as implied by (25), by setting $\mu_{dH}^b = 1$. The steady state spreads between the policy rate and retail deposit rates are calibrated by adjusting the spread parameters μ_{dH}^h and μ_{dH}^f. The average spread between the US policy rate and the effective interest rate on household checking accounts (obtained from FDIC data) over the period 1998Q3-2008Q3 equalled around 300 basis points.[12] However, in our model deposits include a much wider range of bank liabilities, including wholesale deposits which attract rates much closer to the policy rate. To approximate the average convenience yield of total financial sector liabilities to non-banks, we therefore calibrate this spread at 150 basis points. This is very similar to Ashcraft and Steindel (2008), who compute, for the single year 2006, a spread of 134 basis points between the average rate of US commercial banks' portfolio of treasury and agency securities on the one hand and the average rate on their complete portfolio of liabilities on the other hand.

The third subset of parameters determines the size and composition of balance sheets, which are summarized in Figure 3. Retail loans to domestic and foreign households are calibrated at 160% of GDP by adjusting the money demand parameter A^c. This is slightly larger than the ratio of loans of the US commercial banking sector to GDP prior to the GFC. We calibrate the ratio of the value of capital to GDP, which can be shown, based on Fed and NIPA data, to equal around 240% of GDP, by adjusting the steady-state willingness-to-lend parameter for domestic currency loans $\bar{\kappa}_H^h$. The ratio to GDP of foreign currency retail deposits held by domestic households is set to 20% by adjusting the currency home bias parameter b^o, and the ratio to GDP of foreign currency retail loans owed by domestic households is set to a matching 20% by adjusting the parameter φ_b of the interbank money demand function. The initial steady state ratio of financial investor wholesale deposits to GDP is set to 50% of GDP, by adjusting the discount factor of financial investors β_{fi}. We set the ratio of foreign currency cross-border gross interbank deposits to GDP to 20% by adjusting the parameter ϑ_b of the interbank money demand function, with FXMR ensuring the foreign currency gross interbank loans also equal 20% of GDP. The same assumption for the foreign economy, together with FXMR, ensures that the ratios of domestic currency gross interbank deposits and loans also equal 20%.[13] The equal currency split is roughly representative of European banking systems, while the US banking system's interbank balances are almost exclusively in terms of US dollars.

[11] As a cross-check, the average BAA-AAA spread from Moody's over the same period equals 108 basis points. The discrepancy is due to the fact that the average firm has a worse rating than BAA.

[12] This spread has been significantly compressed during the ZLB period, but we exclude this period because we do not consider it representative of normal conditions in the banking sector.

[13] Figure 7 in Lane and Milesi-Ferretti (2018) shows that 20% corresponds approximately to the global average across BIS-reporting banks.

The elasticity of household consumption with respect to total liquidity is calibrated by adjusting the parameter $\mathfrak{m}$. To obtain an approximate guidance for this elasticity, we note that the elasticity of GDP with respect to total liquidity, measured as in our model as a broad aggregate of financial sector liabilities, must necessarily be very close to the elasticity of GDP with respect to financial sector credit. This elasticity equals $(\partial gdp/\partial credit)\,(credit/gdp)$. In the US, the private credit to GDP ratio has recently been near 200%, while the derivative of GDP with respect to private credit, or the additional dollars of GDP generated by an additional dollar of credit, is sometimes referred to as the marginal productivity of debt[14], and has in recent years been around a historical low of around 0.25. We therefore calibrate $\epsilon_c^{hh} = 0.5$, which yields a value of $\mathfrak{m}$ just above 10. Within a broad range of values around $\epsilon_c^{hh} = 0.5$ the properties of the model are not highly sensitive to $\mathfrak{m}$.

The interest semi-elasticity of financial investors' demand for wholesale deposits is calibrated at a very high $\epsilon_d^{fi} = 250$, by adjusting their porfolio preference parameter ϑ. This ensures that financial investors are willing to absorb very large increases in their wholesale deposits for only very small increases, measured in a few basis points, in the wholesale deposit rate relative to the policy rate. Because financial investors function as market makers for CBDC, the issuance of a large stock of CBDC requires that they are willing to exchange a large stock of government debt in their portfolio against a large stock of wholesale deposits without moving prices too much. Our calibration ensures this.

4.3. CBDC-Specific Calibration Issues

The elasticity of substitution between domestic and foreign currency liquidity θ_o is calibrated at a baseline value of 1.5. There is to our knowledge no established literature on this parameter, which is also used in Cesa-Bianchi et al. (2019). Kumhof et al. (2020) conduct sensitivity analysis to explore the sensitivity of their results to θ_o. There is even less established literature on the elasticity of substitution between bank deposits and CBDC θ_d. The reason is, of course, that we have not yet seen a major economy introducing CBDC at scale alongside bank deposits, so that data on this magnitude do not exist. There are some data on the stickiness, or lack of responsiveness to interest rate differentials, of deposits held at different financial institutions, and these suggest that a very high elasticity would not be a realistic assumption, and clearly an assumption of fixed proportions is equally unrealistic. Therefore, while any quantitative experiment that explores the consequences of CBDC introduction needs to make some educated guesses, we can work over a reasonable range and then conduct sensitivity analysis.

Specifically, we have proceeded as follows: To avoid degenerate solutions for the initial steady state, the separable version of the LGF is needed to study the introduction of CBDC into an economy that up to that point used only bank deposits as its medium of exchange. That separable CES LGF features a constant elasticity of substitution but decreasing returns to scale. We argue that initially decreasing returns to scale plus an initially high elasticity of substitution are realistic, and therefore choose $\theta_d = 5$ for the transition simulation. The substitutability between CBDC and deposits is initially likely to be higher because CBDC would initially likely be introduced into markets where it is most competitive with deposits. And returns to scale are initially likely to be higher and later decreasing for the same reason, as during the initial introduction the quantity of CBDC would still be limited and any CBDC that does exist is likely to be at a premium before the market becomes saturated. When we study the economy around a steady state where CBDC is already present, we

[14] For an example, see http://csinvesting.org/2018/03/01/the-falling-marginal-productivity-of-debt/.

start to use the nonseparable LGF, with constant returns to scale as CBDC is now assumed to be in adequate supply, and with a lower elasticity of substitution of $\theta_d = 2$ as CBDC is now competing increasingly in the less competitive retail markets. We have conducted extensive sensitivity analysis to support this set of assumptions. Specifically, for several standard shocks we compared impulse responses for the separable LGF with $\theta_d = 5$ and the nonseparable LGF with $\theta_d = 2$. We found that the differences in aggregate liquidity, and therefore also in monetary transaction costs and real variables, are very small. There are of course differences in how the composition of aggregate liquidity between CBDC and bank deposits changes, but even these are not large, and not nearly large enough to change the signs of the main balance sheet components.

In the transition simulation, with a few exceptions that we will now list, all parameters are identical to those of the pre-CBDC economy. We assume that the steady-state ratio of CBDC to nominal GDP jumps from 0% to 30% in Home, and because CBDC is issued against government debt, the ratio of government debt to GDP jumps from 80% to 50%. These changes are implemented by adjusting $\overline{gd^{rat}}$j and $\bar{m}^{rat}$. No CBDC is issued in Foreign, and government debt stays at 80% of GDP. Because the introduction of CBDC entails significant changes in bank balance sheets, we also allow the dividend payout parameters δ^b to adjust to maintain the steady state capital adequacy ratios at 15.5%. Finally, by adjusting the parameter $\eth$ we impose that in Home the steady state interest rate on CBDC settles at 50 basis points below the interest rate on retail bank deposits due to a higher convenience yield. This requires a relatively modest $\eth$ of 1.048. For the CBDC quantity rule used in the transition simulation we set $m_\pi = 0$, so that the quantity of CBDC is held constant relative to GDP.

In the transition simulations, with a few exceptions that we will once again list, parameters still remain largely identical to those of the pre-CBDC economy. However, in this case the two economies are symmetric, with both featuring a 50% government debt to GDP ratio and a 30% CBDC to GDP ratio. Due to the reductions in government debt relative to the original steady state, real interest rates are now 60 basis points lower in steady state in both economies. We again allow the dividend payout parameters δ^b to adjust to maintain banks' steady state capital adequacy ratios at 15.5%, and we again allow $\eth$ to adjust to ensure that both countries' CBDC interest rates settle at 50 basis points below their respective interest rate on retail bank deposits. The economy with nonseparable LGF features 2 new parameters per country, the share parameters in the LGF b_X^m. We adjust these to fix the ratios to GDP of retail bank deposits at the same levels that they would reach if an economy with separable LGF transitioned to government debt and CBDC levels equal to 50% and 30% of GDP, respectively. We also allow $\mathfrak{m}$ to adjust to main ϵ_c^{hh} at the level that this equivalent economy with separable LGF would reach (this requires only very small adjustments). Finally, we allow ι to adjust to maintain equality of steady state consumption of households and financial investors.

5. Simulation Results

5.1. Transition to an Economy with CBDC

Figure 4 studies the effects of introducing, in period 0, CBDC equal to 30% of GDP, through a purchase of government bonds of the same value, at market-clearing prices. This policy increases GDP by 3.3% in the long run. The completion of the transition to this new steady state takes well over two decades, as the capital stock takes time to reach a new much higher level. Figure 1 studies the first 20 years, or 80 quarters. To ensure that the final steady state values are visible, the red dotted line in each case displays the change, in period 0, of the long-run steady state of the respective variable, while the solid line shows the actual transition path. For CBDC, we assume that its supply is kept at 30% of GDP through a quantity rule (**??**) with $m_\pi = 0$. The long-run net beneficial effects of this CBDC issuance are driven by three main factors, reductions in real interest rates, reductions in distortionary fiscal tax rates, and reductions in monetary distortions.

We begin with interest rate effects. Our calibration of $\phi_f = 0.00005$ implies that a 30 percentage points drop in the ratio of defaultable government debt to GDP is associated with a 60 basis points drop in the real policy rate, from 3% initially to 2.4% in the long run, in Home. The corresponding wholesale deposit rate is determined by financial investors, due to their high interest semi-elasticity. They hold deposits equal to 50% of GDP immediately before the introduction of CBDC, 80% of GDP immediately thereafter as they trade government debt against CBDC with the government and CBDC against bank deposits with households, and just under 93% of GDP in the very long run as they accumulate additional financial assets. Financial investors therefore eventually experience a percent increase in their deposit holdings of 85%. With $\varepsilon_d^{fi} = 250$, the wholesale deposit interest rate must therefore rise by around 17 basis points relative to the policy rate, in other words it drops from 2.75% to 2.32%. An interpretation is that banks' average funding cost increases relative to the policy rate because they have to rely more on wholesale funding, with a sizeable share of retail monetary transaction services now being performed by CBDC instead. Retail deposit rates remain at a 1.25% discount relative to the wholesale deposit rate, at 1.07%. The 50 basis points post-transition steady state real interest rate discount of the CBDC rate relative to the deposit rate is calibrated, and implies a long-run level of the real CBDC rate of 0.57%.

Bank lending and bank deposits increase by around 10% of GDP in the long run[15], as banks satisfy a higher demand for deposit balances. The latter is due to a combination of increased economic activity that requires additional transactions balances, and of the increase in CBDC balances, which requires at least some increase in deposit balances due to imperfect substitutability. Note that due to the high assumed elasticity of substitution between deposits and CBDC, the second effect is not in fact very strong. This result is important because it addresses a common fear concerning the introduction of CBDC, that it might take business away from banks.

Despite the increase in lending, the domestic real wholesale lending rate declines from 3.66% to 3.23% in the long run, and thus follows the wholesale deposit rate almost exactly, with no significant increase in the wholesale lending spread that represents the opportunity cost of bank credit to households. The reason is that the increase in bank lending is not accompanied by a significant increase in bank riskiness. The domestic retail lending rate however declines by only around 15 basis points, from 5.33% to 5.18%, with the 28 basis points increase in the retail lending spread

[15] Loans and deposits drop relative to GDP on impact, but their absolute drop is much smaller.

reflecting higher loan-to-value ratios among bank borrowers. This effect is partly moderated by the fact that the real value of collateral increases due to lower real interest rates. The reductions in real interest rates, by arbitrage with the return to physical capital, which drops from 5.13% to 4.87% in the long run, directly stimulate additional physical capital accumulation and thereby output.

But lower real interest rates also have additional powerful effects through fiscal and liquidity channels. We begin with the former. Following the transition, interest charges on 62.5% of government financing (50% of GDP) drop from 3.0% to 2.4% p.a., while interest charges on the remaining 37.5% (30% of GDP) drop further from 2.4% to 0.57% p.a. The combined budgetary effect of these two savings in financing costs adds up to over 1% of GDP. With the sum of government debt and CBDC remaining constant at 80% of GDP, this means that the long-run ratio of tax revenue to GDP can fall by the same amount. The government's use of these gains to fund reductions in distortionary tax rates further stimulates economic activity. Because tax rates start from different initial levels and change proportionally, the labour income tax rate drops by 106 basis points in the long run, and the capital and consumption tax rates by 114 and 17 basis points.

The final factor, increases of liquidity or monetary transaction balances, cannot be easily isolated from, and instead works synergistically with, real interest rate and tax rate effects. First, part of the gains from lower distortionary fiscal tax rates can be attributed to the interest savings on CBDC, which are due to the liquidity benefits of CBDC. Second, part of the gains from lower real interest rates are due to lower monetary distortions (which are resource-consuming, with $\mathfrak{r} > 0$) rather than lower real interest rates per se.

The Friedman rule (Friedman (1969)) states that, because the marginal cost of producing money in a world of exogenously created high-powered money equals zero, the money supply should if possible be expanded to the point where the marginal benefit of money also equals zero. However, in a world where almost all money is created endogenously by the private banking system, the marginal cost of money creation equals the spread between wholesale loan and deposit rates, which must always remain positive because of financial frictions and financial regulation. The introduction of CBDC, which is created independently of the banking system, allows the economy to avoid part of these frictions and get closer to the Friedman rule, and this explains some of the beneficial effects of CBDC.

The overall long-run output effect in Figure 1 is a very substantial GDP gain of 3.3%, with the consumption gain at 2.2% and the investment gain at almost 7%. Liquidity increases strongly, not only due to the CBDC increase from 0% to 30% of GDP, with $\mathfrak{d} > 1$, but also due to an almost 10% increase in bank deposits in the long run.

In the shorter run, we observe a sizeable and persistent increase in inflation immediately after the introduction of CBDC. The main reason is that the output gains are only realized after a prolonged transition, while aggregate demand picks up much more quickly due to the immediate realization of the associated wealth effects. With demand running ahead of supply, inflation rises, and with the policy rate reacting to inflation, this means that all real interest rates remain elevated for some time. Because continued high real interest rates also imply continued high government financing costs, distortionary taxes have to temporarily remain above their lower long-run level in order to satisfy the fiscal rule. This of course further dampens activity in the short run, relative to the final steady state. Despite this, both consumption and investment immediately start to expand strongly. The reason is an immediate, strong and persistent drop in liquidity tax rates, which is due both to the direct injection of liquidity through CBDC and the later creation of additional bank deposits.

5.2. Gross Inflow into Domestic Currency - Post-CBDC versus Pre-CBDC

5.2.1. Post-CBDC: Detailed Analysis

Figure 5 studies a shock that increases Foreign households' demand for Home currency deposits. The shock consists of a large and persistent ($\rho^*_{mm} = 0.95$) decline in the home bias parameter $S^{mm^*}_t$ in the Foreign liquidity aggregate LIQ^*_t. From the perspective of Foreign, this represents a sudden shift in domestic investor preferences that could be interpreted as capital flight to the safety of the US dollar. From the perspective of Home, this is the most natural representation of a "capital inflow shock". However, such a financial inflow is automatically a gross flow with zero net value, because it must have as its instantaneous counterpart a matching financial outflow. In other words, the current account and net foreign asset effects of such a shock must equal zero on impact, and any current account effects that do occur are indirect, through the effects of changes in relative currency demands on the exchange rate and thereby on exports and imports.

Responses in the post-CBDC economy, under the assumption that both central banks are pursuing interest rate rules for CBDC, are shown as black solid lines in Figure 5. Because the FXMR rule prohibits a change in the net balance between both Home and Foreign banks' foreign currency nostro and vostro accounts, all adjustments to the gross deposit inflow have to be made through changes in household loans or exchanges of household deposits. Price adjustments play a critical role in these adjustments. First, prices need to change to make it optimal for Home banks to offer additional Home currency credit, albeit not at the level demanded at pre-existing prices and interest rates. Second, they need to change to make it optimal for Foreign banks to reduce Foreign currency credit. Third, they need to change to make it optimal for Home households to reduce their desired holdings of Home currency deposits so that they are willing to exchange them against Foreign currency deposits with Foreign households. Home households can offer Foreign households either some of their existing Home currency deposits, or Home currency deposits that they obtain by themselves obtaining new Home currency credit from Home banks. Fourth, they need to change to make it optimal for Foreign households to reduce the desired increase in their Home currency deposits to the point where their demand can be satisfied by either Home banks' credit or Home households' deposits.

There are several reasons why the ability of Home banks to satisfy the additional Foreign demand for Home currency deposits through additional loans is limited. First, the steady state Foreign collateral share $\bar{\kappa}^f_H$ that is available to secure such loans is far too small to collateralize a large increase in loans. Second, for banks additional lending has not only a regulatory cost but also a cost due to a tightening of their participation constraint. The latter is affected by changes in the value of land, which decreases in Foreign in Foreign currency. The reason is that a relative decrease in demand for loans in Foreign currency, which are disproportionately collateralized by Foreign land, decreases demand for that land, and therefore, given fixed supply, decreases its price. Third, the value of Foreign collateral in Home currency drops due to the real exchange rate appreciation, which will be discussed below.

In quantitative terms, and focusing on the impact period, Home banks increase Home currency loans by 3% of Home GDP, divided into increases in loans to Home households, by 2.4% of GDP, and to Foreign households, by 0.6% of GDP. Foreign banks similarly decrease Foreign currency loans, divided into decreases in loans to Foreign and Home households. Because the responses of Home and Foreign loan supplies are limited by the lending technology and the availability of collateral, they create and destroy deposits in the desired currencies, but not in sufficient quantities, and not for the households who require them. A further reallocation of deposits across countries and households is therefore necessary.

To study this reallocation we turn to the MUIP spread. The ceteris paribus effect of the decrease in Foreign households' preference parameter $S_t^{mm^*}$ is an increase in their MUIP spread. MUIP spreads need to be arbitraged worldwide, so that the increase in the Foreign MUIP spread must be matched by a decrease in the Home MUIP spread, which is the spread shown in Figure 5. This represents an increase in the relative convenience yield of Home currency, and therefore a decrease in the required financial yield of Home currency. In the case of Foreign households this is a direct result of their increase in preferences for Home currency. Home households do not experience this preference-driven change in convenience yields. Instead they respond only to the change in relative financial yields in favor of Foreign currency deposits. They do so by increasing their relative holdings of Foreign currency deposits at the expense of Home currency deposits, through an exchange with Foreign households. This brings their relative convenience yield into line with that of Foreign households. The exchange of deposits satisfies a major portion of Foreign households' increased demand for Home currency, and this is why loans do not need to change one-for-one to accommodate the change in currency preferences.

Because policy rates are set with inertia by policymakers, changes in policy rates accomplish only a small part of the required relative changes in financial yields, with the major part accomplished by a large expected rate of real exchange rate depreciation of the Home currency. Given an unchanged steady state real exchange rate, on impact the Home real exchange rate therefore appreciates by more than 3%. This is an intuitive result - an increase in demand for Home currency leads to its appreciation. The appreciation is expansionary and inflationary for the Foreign economy and contractionary and disinflationary for the Home economy. This calls for a persistent decrease in the Home policy rate and a persistent increase in the Foreign policy rate, following initial movements of real rates in the opposite direction because of interest rate smoothing. This explains why policy rates move in the right direction to contribute to the decrease in the Home MUIP spread.

In quantitative terms, and again focusing on the impact period and for now only on deposits, for Foreign households the increase in the relative financial return on Foreign currency serves to moderate the increase in their demand for Home currency deposits, which nevertheless equals over 4% of Home GDP, and also the decrease in their demand for Foreign currency deposits. For Home households the same change in relative financial returns leads them to increase their holdings of Foreign currency deposits by around 4% of Home GDP, and to decrease their holdings of Home currency deposits by around 2.5% of Home GDP. Therefore, of Foreign households' 4% of Home GDP increase in demand for Home currency deposits, only 0.6% of GDP is satisfied by additional Home currency loans to Foreign households, with the remainder coming from an exchange of Home currency deposits against Foreign currency deposits with Home households, who obtain additional Home currency loans but end up holding a reduced stock of Home currency deposits. For Home banks, who experience a 4% of GDP increase in demand for Home currency deposits by Foreign households and also an over 1% of GDP increase in demand by domestic financial investors, a combination of an approximately 3% of GDP increase in Home currency loans and an over 2% of GDP decrease in Home currency deposits by Home households allows Home banks to match their Home currency books, and thus also their Foreign currency books. A similar logic holds for Foreign banks.

For interbank reserves, the large transfer of Home household deposits in Home banks to deposits in Foreign banks leads to reserve losses for Home banks that cancel out the reserve gains that came with the original deposits by Foreign households. In aggregate, because mismatched interbank positions in either currency are ruled out, the quantity of domestic currency lending, which is not infinitely elastic, directly determines the quantity of domestic currency deposits. Returns adjust in order to moderate the increase in demand for Home currency deposits by Foreign households, and by decreasing the demand for Home currency deposits by Home households.

Turning to the real economy, the cut in the policy rate and the wealth effect of the appreciating real exchange rate stimulate Home consumption and investment. This deteriorates the current account and thereby leads to an approximately 0.2% decrease in Home GDP. This is therefore an important example of a purely financial shock that, through the real exchange rate, has effects, albeit quite small, on the real economy. The size of this effect depends on the details of the calibration of the nonfinancial side of the economy.

We discuss the CBDC effects last, because this offers a natural transition to the next subsection. We observe that the various inflows into and outflows from bank deposits are matched by inflows into and outflows from CBDC accounts in the same currency. Both signs and magnitudes are similar, with inflows into bank deposits somewhat larger relative to GDP because they account for a larger steady state share of liquidity. Home households are willing to exchange some of their Home currency CBDC for Foreign currency CBDC, for the same reasons as discussed for bank deposits above. And the Home central bank, under its interest rate rule, is willing to supply some additional CBDC in exchange for government bonds, similar to the increased supply of bank credit by Home banks in the case of bank deposits. As a result, financial investors end up holding fewer government bonds and more wholesale bank deposits.

5.2.2. Comparison of Pre-CBDC to Post-CBDC

Responses in the pre-CBDC economy are shown as red dashed lines in Figure 5. In this case there is no CBDC, and the full effect of the shock is felt in the demand for bank deposits alone. As a result, bank deposits are far more volatile, and so is investment and, to a lesser extent, consumption. At the same time, the volatility in the real exchange rate and the current account are reduced, but only very slightly. The main effect of the presence of CBDC is therefore to reduce, rather than increase, the financial sector volatility resulting from volatile capital flows. This is an additional benefit of CBDC issuance that is, of course, not present in a closed economy model.

5.3. Global Runs into CBDC and Financial Sector Stability

Figure 6 studies a set of shocks, symmetric across Home and Foreign, that increases both Home and Foreign households' demand for Home and Foreign currency CBDC, at the expense of their respective holdings of bank deposits in the same currency. Both economies' central banks pursue interest rate rules for CBDC. This is an extension to the international dimension of the discussion of risks posed by CBDC to the stability of the banking system, specifically run risks.

As argued in Kumhof and Noone (2021), run risks are over-emphasized, because appropriate issuance arrangements for CBDC can keep them very manageable. The reason is that they imply that a run into CBDC is a run from eligible assets, specifically from government bonds, and not a run from bank deposits. As discussed above, the issuance arrangements proposed by Kumhof and Noone (2021) are part and parcel of our model. We will now demonstrate how they ensure a smooth response to a run into CBDC.

We start by noting that this is a purely financial shock that is furthermore symmetric across countries. As a result, there are no effects on the exchange rate and the current account, and only negligible effects on GDP. The main effect is therefore on the financial sector. And here we observe, by comparing the second and third columns of Figure 6, that all four household holdings of CBDC increase, and the corresponding holdings of household bank deposits decrease. However, the large increase in demand for CBDC triggers a large sell-off, equal to almost 10% of GDP, of government bonds by financial investors, which is mirrored by an increase in household CBDC holdings but also by an increase in financial investor wholesale deposit holdings. In fact, their increase in deposit demand more than offsets households' decrease in deposit demand, to the point that overall deposits, and therefore also loans, increase by 2% of GDP. In other words, a run from bank deposits into CBDC increases the size of bank balance sheets, albeit with an increased bank reliance on more expensive wholesale rather than retail funding. If this shock persisted, it could have problematic effects for banks' profitability, but the effects on financial stability, meaning here the continued availability of bank funding, are benign or even positive. The reason is that under our issuance arrangements a run into CBDC is not a run from bank deposits, it is a run from the assets that agents need to provide to obtain additional CBDC, government bonds.

5.4. Runs into Home CBDC and away from Foreign CBDC

Figure 7 studies a set of shocks that increase both Home and Foreign households' demand for Home currency CBDC at the expense of Home currency deposits, and that increase their demand for Foreign currency deposits at the expense of Foreign currency CBDC. This studies the risks posed by a dominant economy's CBDC to the stability of its banking system.

The black solid line in Figure 7 represents the case where both countries' central banks pursue CBDC interest rate rules. In this case the demand for Home currency CBDC, equal to almost 15% of GDP, is flexibly met by the central bank with the help of financial investors, who as in a similar case above increase their wholesale deposits to such an extent that the Home banking sector's balance sheet in fact grows, despite the movement of many retail funds to CBDC. The same story in reverse is observed in Foreign. The flexible response of the two central banks ensures that these major financial shocks have almost no real consequences, with all real variables, and also the exchange rate and the current account, moving little. The movements that we do observe are expansionary, because the large and sustained reduction in government debt contributes to a reduction in real policy rates that stimulates investment.

The red dotted line in Figure 7 represents the case where the Foreign central bank pursues an interest rate rule but the Home central bank pursues a rigid quantity rule. In this case, the demand for Home currency CBDC is not satisfied, and the effects of the increased demand for Home currency CBDC are instead seen in an approximately 250 basis point drop in the interest rate on CBDC, into deeply negative territory. This endogenously chokes off the increase in demand for Home currency CBDC, and as a result Home currency bank deposits, both of households and financial investors, also barely move. But in this case Home households are forced to satisfy their liquidity needs with the bank deposits that they are trying to offload rather than use for purchases. As a result total liquidity contracts sharply, and so does consumption, which is also negatively affected by a real policy rate that no longer declines in the absence of a sustained reduction in government debt. The effect on GDP is however buffered by a real depreciation that improves the current account. These results of course have echoes of Poole (1970), who argued that in the presence of significant money demand shocks a rigid money supply rule makes output more volatile. In our environment this only applies to CBDC and shocks that specifically change the demand for CBDC, which represents only a portion of overall liquidity. Banks remain willing to flexibly supply liquidity when there is increased demand for bank deposits. Nevertheless, the logic of Poole (1970) is clearly visible in Figure 7.

5.5. Credit Booms and the CBDC Interest Rate as a Second Policy Tool

Figure 8 studies changes in the ability of countercyclical monetary policy to stabilize output and inflation in the presence of CBDC. The economy is subjected to a change in Home banks' willingness to lend to both Home and Foreign households, which results in large increases in Home currency loans and (not shown) drops in Home currency wholesale lending spreads. Home currency deposits increase, while there is also some decrease in Foreign currency deposits, as households are able to satisfy more of their liquidity needs using Home currency. Because the provision of additional credit increases total liquidity and thereby reduces monetary transacations costs, consumption and output are stimulated.

The solid line in Figure 8 represents a CBDC interest rate rule with $m_\pi = 0$, in other words without an additional countercyclical response to inflation. The blue dashed line represents $m_\pi = 1$, and the red dotted line $m_\pi = 4$. With more aggressively countercyclical rule, the CBDC interest rate is reduced more aggressively in response to higher inflation, and as a result the quantity of CBDC decreases more rapidly. This in turn increasingly offsets the increase in total liquidity brought about by the increase in bank deposits, and stabilizes the effective price of consumption. Countercyclical CBDC policy thereby helps to reduce the volatility of consumption, investment, output and inflation. In other words, the CBDC interest rate can be used synergistically with the traditional policy rate to stabilize output and inflation, with the policy rate working primarily through real channels like intertemporal substitution in consumption and investment, and the CBDC rate working primarily through financial channels like liquidity, money velocity, and the effective price of consumption.

6. Conclusions

This paper studies the open-economy implications of introducing CBDCs into a 2-country DSGE environment that features a realistic financial system. Households derive liquidity services from a combination of bank deposits and CBDCs, which are imperfect substitutes. We focus on retail CBDCs that can be held by foreign as well as domestic households, and that are introduced via central bank purchases of government bonds or transfers to the government budget, ruling out direct and guaranteed conversion of bank deposits into CBDC at commercial banks. We model CBDCs as strictly separated from reserves, and remunerated at an interest rate below the policy rate due to their non-pecuniary convenience yield.

The paper shows that the introduction of a CBDC by a single economy is highly beneficial in terms of output (and welfare; to be shown). The effects of domestic and cross-border financial disturbances are not exacerbated by the presence of CBDCs, in fact the effects on bank balance sheets are typically mitigated. Large reallocations of liquidity between currencies, and between deposits and CBDC, yield benign balance sheet adjustments and small real effects, especially when CBDC is supplied flexibly by the central bank, subject to an interest rate rule and against government bonds. Finally, a more aggressively countercyclical use of the interest rate on CBDC could be highly beneficial in terms of stabilizing output and inflation, because it adds a monetary transmission channel to the traditional intertemporal substitution channel that is used by the tradional policy rate on reserves.

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Figure 1. Domestic and Cross-Border Gross Financial Flows

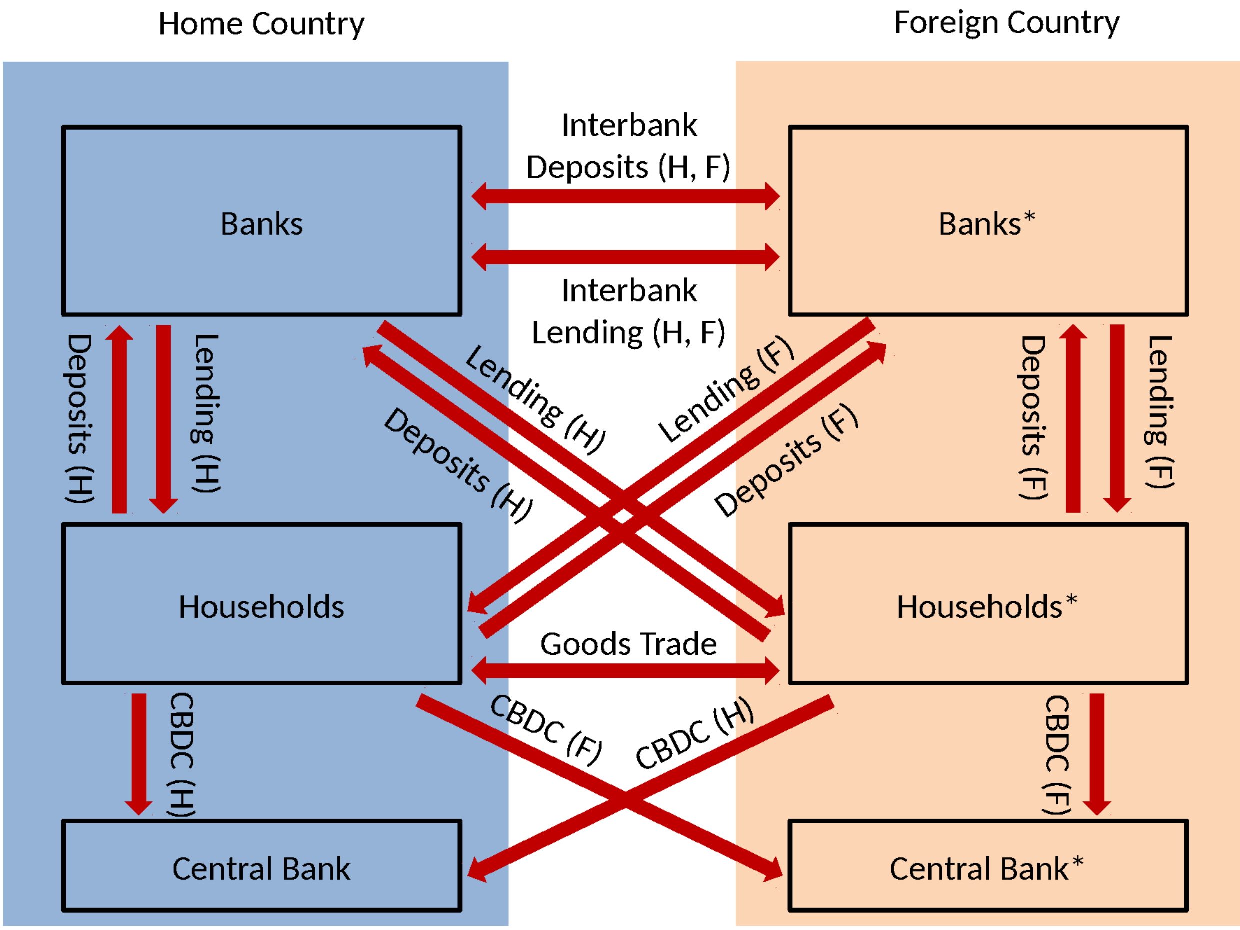

Figure 2. Calibration: Steady State Real Interest Rates

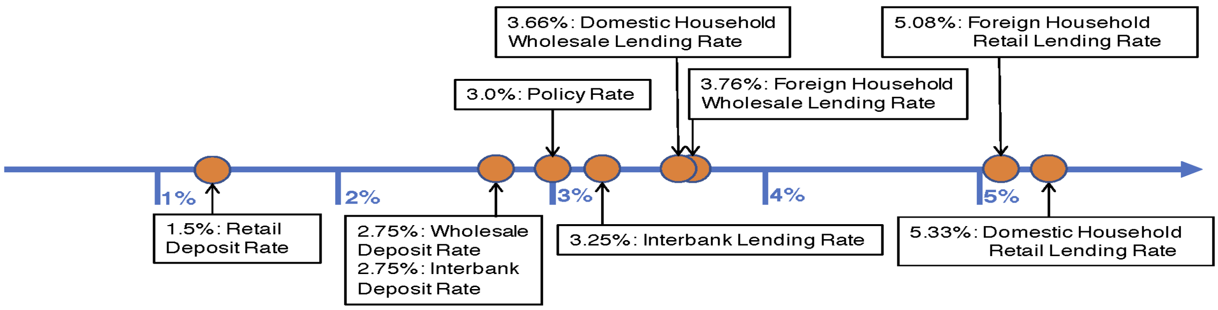

Figure 3. Calibration: Steady State Balance Sheets

Home Banks

Home HH Loans	140	Net Worth	27
		Home FI Deposits	50
		Home HH Deposits	63
Foreign HH Loans	20	Foreign HH Deposits	20
Home Ccy Nostro	20	Home Ccy Vostro	20
Foreign Cccy Nostro	20	Foreign Cccy Nostro	20

Foreign Banks

Foreign HH Loans	140	Net Worth	27
		Foreign FI Deposits	50
		Foreign HH Deposits	63
Home HH Loans	20	Home HH Deposits	20
Foreign Cccy Nostro	20	Foreign Cccy Nostro	20
Home Ccy Nostro	20	Home Ccy Vostro	20

Figure 4. Transition to an Economy with CBDC

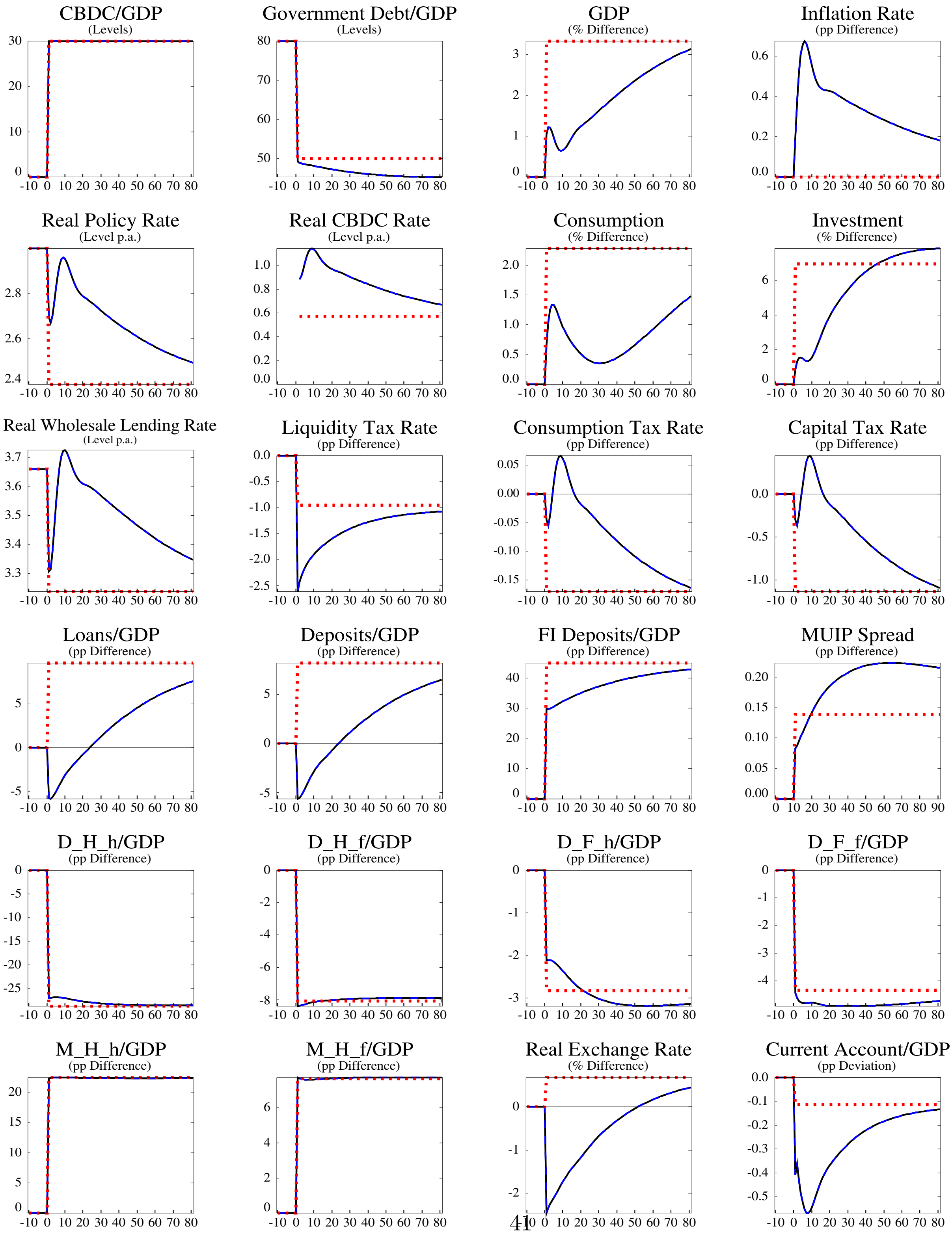

Figure 5. Gross Inflow into Domestic Currency - Post-CBDC versus Pre-CBDC

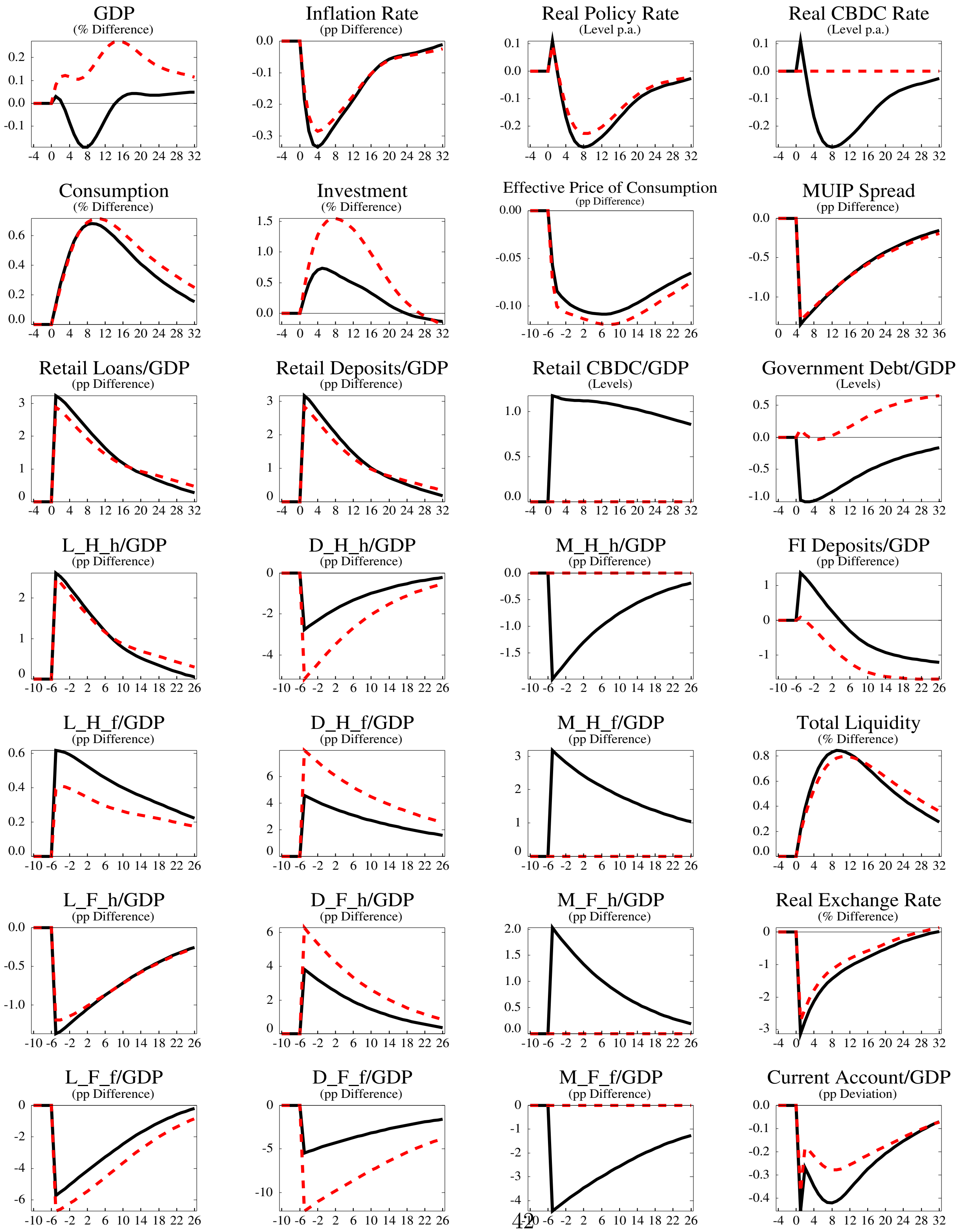

Figure 6. Global Runs into CBDC and Financial Sector Stability

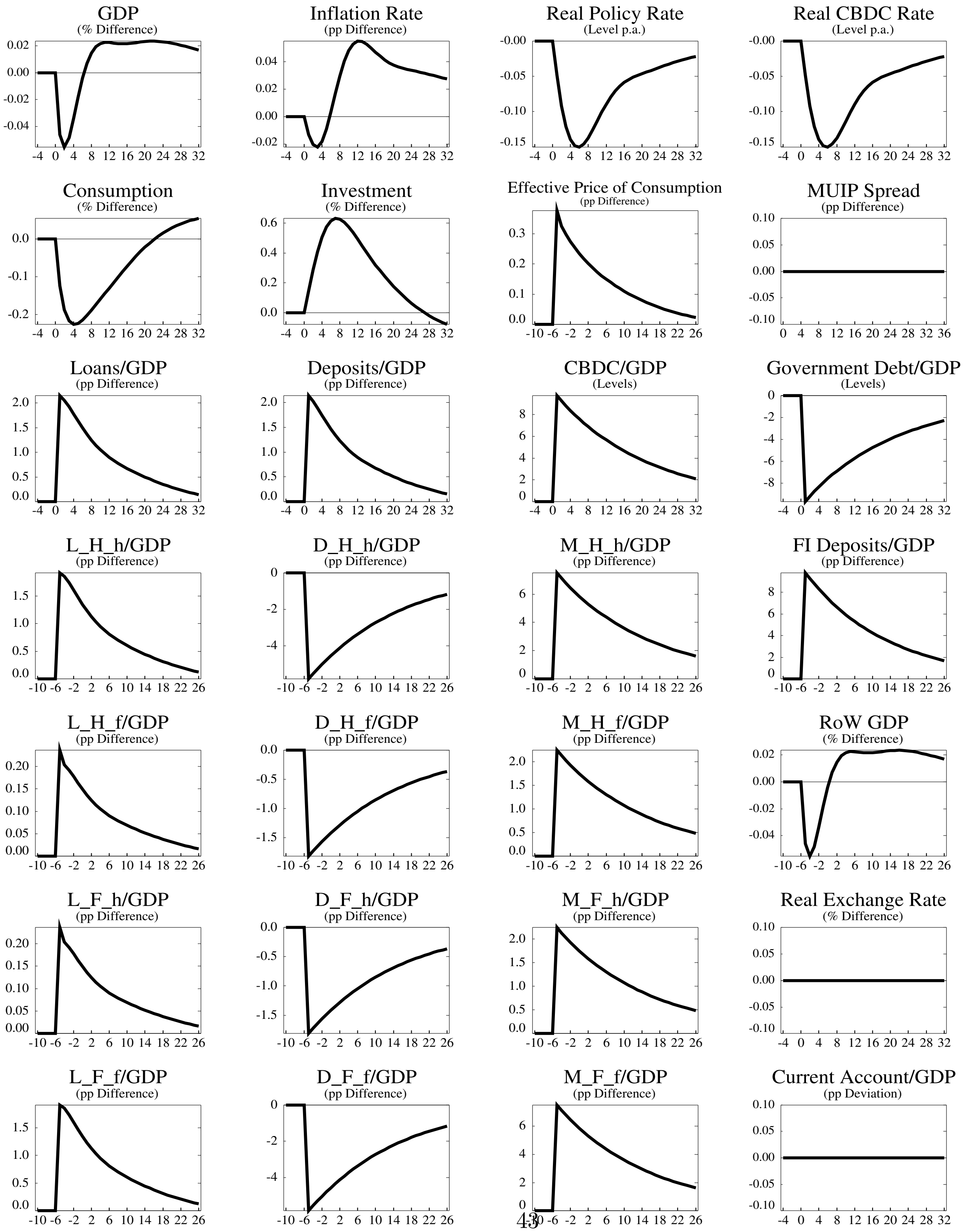

Figure 7. Run into Home Currency CBDC and away from Foreign Currency CBDC

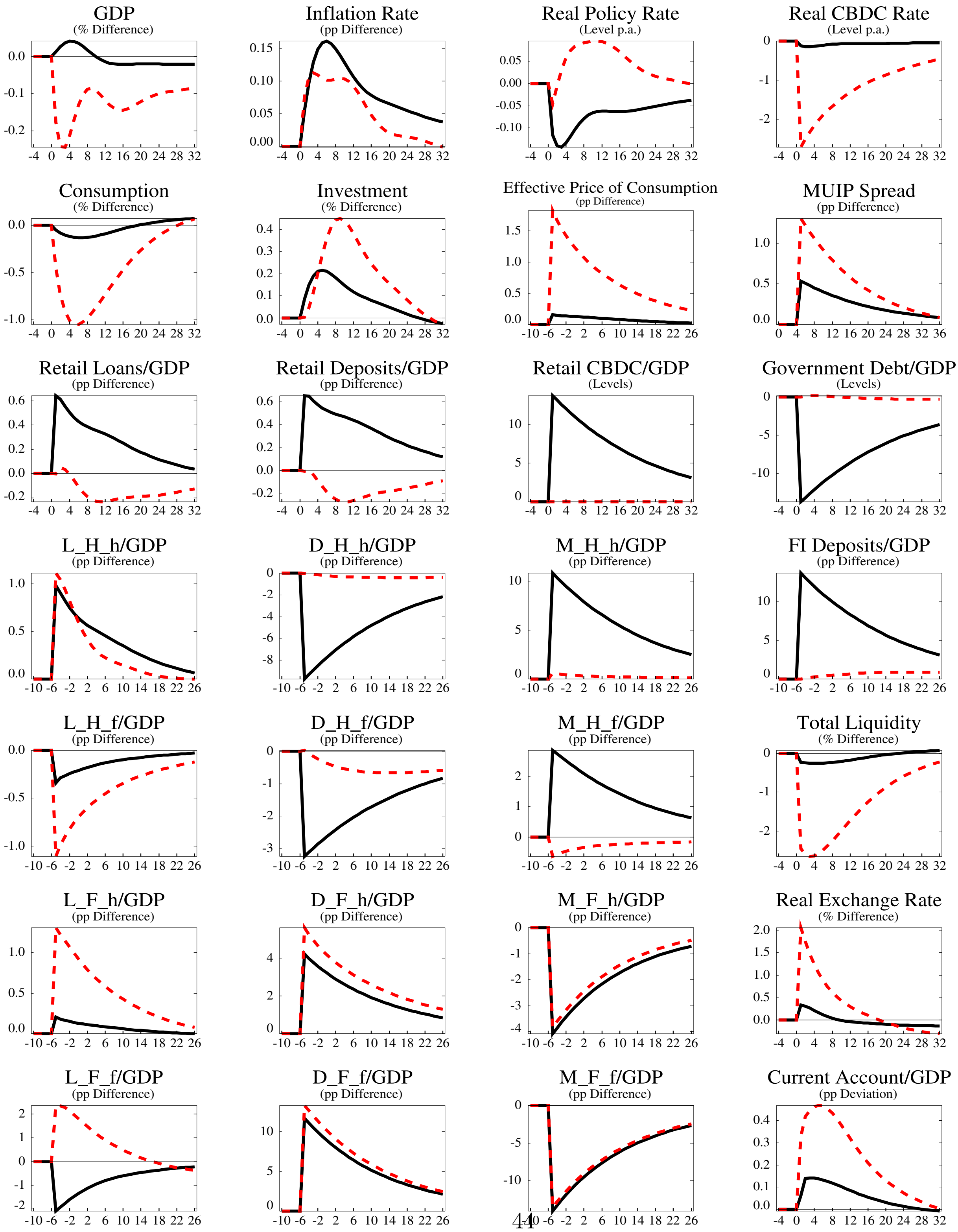

Figure 8. Credit Booms and the CBDC Interest Rate as a Second Policy Tool