Fiscal Governance after the Financial Crisis: A review

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Abstract: Economists have traditionally used a simple rule that restricts primary deficits to less than a threshold given by the interest-growth rate differential and existing debt in order to judge fiscal sustainability. This rule derives from a single period application of the government budget constraint. It is not forward looking. In the equivalent dynamic rule, the primary surplus needs to match any expected, discounted increases in public spending, the net interest on existing debt, and preferences for extending debt relative to changing taxes.

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1 This paper was presented to the Economic Society of Australia, “Symposium on Lessons from the Global Financial Crisis”, Griffith University, 7 March 2019. It reflects the lessons learned from joint research on fiscal governance carried out with my colleagues: Floriana Cerniglia and Enzo Dia. However, I am responsible for the interpretation placed on the results which emerged.
1. Introduction

All the advanced OECD economies were adversely affected by the Great Financial Crisis, although some like Australia and Canada were less affected than those in Europe. The common feature seems to have been a marked worsening of their public debt burdens, and an inability to prevent their budget balances from deteriorating during the crisis and (more importantly) an inability to successfully repair their fiscal balances after the crisis.

This paper reviews the rules we currently use, and those currently recommended, for limiting the size of fiscal deficits and preventing the kind of permanent debt increases we have seen over the past years. The principal focus is on which of these rules work, and on explaining why they very often do not work.

The simplest rules (using IMF recommendations) are based on restrictions on public spending, raising sufficient revenues, and/or maintaining strict balanced budget restrictions. But as such, they do not place or guarantee any explicit restraints on the underlying stock of public debt; and they suffer from a general "endogeneity problem" (that is: by placing restrictions on revenues, public spending or the budget balance will automatically change income levels and hence cause us to miss the outcomes previously planned for those target variables).

The next type of rule (currently favoured by the IMF staff) is one designed to keep public debt levels constant - or move them up or down by specified amounts, within the current period – and then to be repeated in following periods as needed (Blanchard, 2019). This rule notes that the change in the debt burden (i.e. the budget surplus) will be the difference between the interest payable on the existing debt burden and the current rate of output growth, multiplied by the current debt burden. Calculating this quantity gives a minimum value for the primary target surplus if we don't want debt to increase. A smaller surplus will allow debt to increase; a larger surplus will force debt to decrease. The difficulties with this rule are:
1. It implicitly assumes that the responses of taxes or public spending to the policy changes necessary to get the primary surplus we need, are infinitely and instantaneously elastic. This is most unlikely to be the case in practice.

2. You need to make error free, unbiased forecasts of the future interest rate payable on the average stock of debt (relatively easy to calculate) and of future output growth (not so easy to calculate).

3. The growth adjusted interest rate must allow for the possibility of default risk premia and inflation risk premia; and for the fact that the CPI inflation rate to be subtracted from the interest rate will be different from the GDP inflation rate to be subtracted from the growth rate (we need to work in real terms).

4. Sensitivity to forecast or any other errors can easily destabilize the stock of debt, adding to the burden in bad times in a way that cannot be retrieved at a later time.

Numerical simulations of the evolution of debt under this control rule in the US and UK show: i) it has consistently failed to prevent debt growing at different times over the past 50 years; ii) it has failed to bring down debt even when actual primary surpluses have exceeded the threshold values specified by the rule. Hence, this rule is neither sufficient nor necessary to control debt or to ensure public finances remain sustainable.

2. The performance of traditional debt stabilisation rules

2.1 Theory

The standard text-book rule for stabilising public debt requires the government to obtain a primary surplus $S_t$ greater than or equal to the difference between the interest rate currently payable on outstanding debt, $r_t$, and the current rate of growth of nominal GDP $\rho_t$, multiplied by the existing debt to GDP ratio $B_t$: 

$$S_t \geq (r_t - \rho_t)B_t.$$  

(1)

How can a government implement such a rule? When planning the budget for the following year, the government needs to forecast the value of the difference $r_t - \rho_t$. If the government plans to reach a certain target for the debt to GDP ratio, $B_t$, in the future, it needs to plan a sequence of expected surpluses over those years to
arrive at the desired target. In the following period the government needs to verify the results achieved so far, to renew the forecast of the relevant variables in the light of new information, and to define a new set of surpluses. The rule therefore requires a process based on forecasting forward looking data, plus a feedback adjustment for when new information on past failures and on future expected interest rates/growth rate values becomes available:

$$E_{t-1}[S_t] \geq E_{t-1}[r_t - \rho_t]B_t.$$  

(2)

The forecasting error when predicting a sequence of future expected values for the difference $r_t - \rho_t$ will be almost entirely due to GDP shocks since $r_t$ is to a large extent (but not completely) predetermined.\(^2\) Since no country is financed entirely with short-term debt, only a fraction of the outstanding debt in each period will be refinanced at current interest rates. For example, if the average life of outstanding debt is 10 years,\(^3\) on average only one tenth of the outstanding stock of debt needs to be rolled over at the current interest rate and refinanced each year. Correspondingly any change in interest rates will affect only one tenth of the debt stock.

More generally, if debt maturity spans $n$ years, the expected interest rate applicable to the next budget is $r_t = \left(\frac{n-1}{n}\right)\bar{r}_t + \frac{1}{n}E[D_{t+1}]$, where $\bar{r}_t$ is the historical cost of outstanding debt (a weighted average of past interest rates) and $E[D_{t+1}]$ is the expected cost of newly issued debt.

By contrast, since the rate of growth of nominal GDP is the sum of the rate of growth of real GDP and of inflation. Both are typically I(1) series: the government needs to be able to forecast the interaction between two random walk series with drift, $\rho_t = E(g_{t+1}) + E(\pi_{t+1}) + E(g_{t+1} \cdot \pi_{t+1})$, where $g_t$ is the rate of growth of real GDP at time $t$ and $\pi_t$ is the inflation rate at time $t$. Typically, we ignore the last term as second order small although this may not always be true in discrete time applications.

That said, the larger forecasting errors normally occur during recessions since the

\( r_t \) represents the effective rate of interest payable on the stock of current and past debt.

\( For \) example, the average term to maturity of debt in Italy is 6.7 years, whereas that in the United Kingdom is 14 years.
volatility of the series is typically clustered during downturns. These errors can be substantial. During the last recession the output gap was, in most countries, 5 – 7 per cent and the deflation rate was around 2 percent. As a result, the typical value for $\rho_t$ was around 10 percent below trend. To pretend that the growth rate will remain roughly constant in the next period(s) would be a mistake. Interest rates may also not remain constant in recessions. Nevertheless, the interest rate to be applied to newly issued debt, on the average cost of debt, is rather small in most countries compared to errors/variations in the growth rate.

Yet, even if these approximation errors are small, there are further problems with (1). When a recession occurs, any debt reduction plan may be suspected of being not feasible or not credible. Second, the stabilisation rule, (1), will require a massive dose of austerity which makes the recession deeper. The government is then forced to choose between debt stabilisation that risks spiraling into an even worse recession, or fiscal policy requiring more debt to reduce the output gap. Any government may be expected to choose the latter path for economic and political reasons, so long as financial markets or international institutions are willing to provide credit. Either case will attract additional risk premia - making existing and future debt less sustainable than previously predicted.

Alternatively, a government whose fiscal space is sufficiently large may choose to adopt an expansive fiscal policy while keeping to a long term debt target, by extending the stabilisation plan over a longer horizon. However, since any debt could be accommodated that way, it may become difficult to commit to any specific path and the available fiscal space may become tight for countries already holding large amounts of debt. In practice, if the government cannot inflate its debt away to achieve its target because debt is denominated in foreign currency, or because the country has no control over monetary policy, the maturity extension exercise has to become coercive through some form of default.

With these comments in mind, it is worthwhile to review what the traditional stabilisation rule calculates. At each $t$, the existing stock of debt, $B_t$, is known. But
the growth adjusted interest rate is more complicated. In nominal terms, to match how debt is recorded in national accounts, (1) contains:

\[ E_t(r_t) - E_t(\rho_t) = \sum_{j=1}^{T} w_{t-j} r_{t-j} + w_t E_t(r_t) - E_t(\rho_{t+1}) \]  

(3)

where \( T \) represents the oldest maturity held, and \( w_t \) is the proportion of maturity \( t \) in total debt. Hence evaluated in real terms to reflect how the final outcomes for consumption, investment, and the debt to GDP ratio will react to different levels of debt, the corresponding term is:

\[ E_t(\hat{r}_t) - E_t(g_t) = \sum_{j=1}^{T} w_{t-j} (r_{t-j} - \pi_{t-j}) + w_t E_t(r_t - \pi_t) - E_t(\rho_{t+1}) + E_t(\pi_{t+1}) \]  

(4)

\[ \neq \sum_{j=1}^{T} w_{t-j} r_{t-j} - E_t(\pi_t) - E_t[\rho_{t+1} - E_t(\pi_{t+1})]. \]  

(5)

where \( \hat{r}_t \) is the real interest rate on debt at \( t \). The lower expression is the growth adjusted interest rate that would be used if the standard debt stabilisation rule is defined in real terms. In the standard framework, inter-temporal optimisation is not necessary because the institutional framework of the models is extremely simple. But when considering a more realistic environment, inter-temporal optimisation will become necessary for a number of reasons:

i) To balance the benefits of raising taxes now against extending debt (and hence raising taxes later).

ii) Because the expected inflation terms do not cancel out between \( r_t \) and \( \rho_t \) as usually assumed when the standard rule is applied.

iii) Because \( r_t, \pi_t \) and \( \rho_t \) are jointly dependent, and also jointly dependent over time.

iv) Because adjustment costs in tax revenues and spending are not zero. So we should expect incomplete adjustments over specific time intervals.

v) Since fiscal policy is not in real time, any budget surplus must be planned in advance on the basis of the available information regarding future variables, including the expected trend in entitlement spending.

Do points i) to v) matter? Fig. 1 illustrates the path that current debt would take if the traditional primary surplus rule was applied consistently in the OECD. To see
the implications, we proceed as follows. The law of motion of the debt ratio is

$$B_{t+1} = B_t + (r_t - \rho_t)B_t - S_t.$$  \hspace{1cm} (6)

Next, we define a similar path for “controlled” debt; that is the path you would get if the traditional primary surplus rule had been applied each year by generating primary surpluses equal to (2). Defining $\tilde{B}_t$ as controlled debt at time $t$ under the standard assumptions that nominal output and interest costs are random walks with drift, we get:

$$E_t(r_{t+1}) = r_t; \quad E_t(\rho_{t+1}) = \rho_t; \quad \text{and} \quad \tilde{B}_{t+1} = \tilde{B}_t + (r_t - \rho_t)\tilde{B}_t + (r_{t-1} - \rho_{t-1})\tilde{B}_t$$  \hspace{1cm} (7)

Since $r_t - r_{t-1} \neq 0$ and $\rho_t - \rho_{t-1} \neq 0$, it now follows that $\tilde{B}_{t+1} \neq \tilde{B}_t$. Hence the standard primary surplus rule does not necessarily produce constant debt; or indeed falling debt if that had been planned. In this world, it is not a reliable way to achieve debt control.

### 2.2 Evidence

Figure 1 displays the behaviour of $\tilde{B}$ for five large OECD economies: the US, UK, Germany, Italy and Japan, from 1960 to 2015. We choose large OECD economies because we want to use a coherent and uniform set of data. However the same logic applies to any country. Fig. 1 suggests that, although the rule would have kept debt inside a reasonably tight band, it would also generate substantial volatility in debt, so much so that in some periods it failed to keep debt on a sustainable or constant path. The diagrams also reveal other relevant features:

i) The volatility induced by GDP shocks is substantial, as recessions cause sharp increases in debt. This exercise actually underestimates the problem because we abstract from the negative feedback caused by the austerity measures induced by the rule. They would make a recession much deeper. In

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4 Instead it would have kept US debt between 63 and 70% with an upward trend; in Italy between 36 and 44% with an upward trend; in the UK between 57% and 70%, an upward trend until 1995; Japan from 135% to 170%, an upward trend, and Germany from 43% to 39%, a downward trend.
addition, the degree of austerity necessary to stabilise debt by abiding by the rule is likely to be large, since the rule would impose very quick fiscal corrections. These policies would not be viewed as feasible or credible.

ii) Many of the downward corrections that we observe are due to unexpected inflation, particularly during the late 1970s. When monetary policy becomes independent, the rule will require larger surpluses.

We can now analyse the actual behaviour of governments by comparing the paths obtained for controlled debt $\bar{B}_t$ with those of actual debt $B_t$; and by comparing the primary surpluses necessary to control debt with the corresponding actual primary surpluses. Debt levels have increased steadily in most countries in recent years, and these rises in debt have almost always been ascribed to a lack of fiscal discipline of the kind required by the traditional rule. We can test that proposition directly by comparing if actual primary surpluses $S_t$ shown above were above or below the necessary threshold $(r_t - \rho_t)B_t$. This is done in Figures 2 and 3 for the United States and United Kingdom respectively.
Figure 1: Controlled debt paths for selected OECD countries

United States

United Kingdom

Italy

Germany

Japan
First, it is of interest that the actual debt levels in the US have been below the controlled debt path for most of the period before 2007, with the exception of the early 1990s and after 2005. But the primary surpluses were below (not above) the required threshold, which explains why debt was on a rising path from 1980 onwards (1995-2000 excepted when primary surpluses were large enough to prevent that). This result is consistent with the evidence of Ardagna et al (2007) and Bohn (2011) suggesting that the US Federal government has adjusted revenues and expenditure to stabilise debt over its history up to 1998.\(^5\)

\(^5\) Jones and Joulfaian (1992) and Bohn (2011) analysed the stability of public debt in the United States by testing the response of the primary surplus to the level of the stock of debt. They find a statistically significant and economically relevant response, using data covering the whole history of the country. Mendoza and Ostry (2008) meanwhile used the
does not fall even when primary surpluses are large enough - implying that the traditional rule was not the correct rule to reduce debt, and that the US government had achieved better results by other means (fig. 2).

Compare the primary surpluses to the threshold necessary to make debt go down: \( S_t > (r_t - \rho_t)B_t \). We find the primary surplus (red) was in deficit and mostly below the threshold (blue) till 1977; equal to it or above to 1980; below it again in 1980 to 1995; then above it for 1996-2001; and clearly below it in 2001-2015. So the US did not keep to the traditional rule very often before 1994, yet debt remained below 1960s levels until 1990. In the Clinton era the Federal government achieved primary surpluses above the threshold from 1996 until 2001, but then spectacularly failed to do so from 2001-2015. However debt remained close to the initial level till 2007. The evidence for the United States therefore suggests that the traditional rule has been virtually irrelevant until 2001.

Now look at the UK, fig 3. Once again controlled debt never falls, which again implies that the traditional rule was not the correct rule to use to reduce debt. In contrast, actual debt fell to 1990 and from 1997-2002 which implies that the UK found an alternative rule (the fiscal deflation and pro-growth policies of the late Thatcher and early Blair regimes). In this case also it shifted upwards in the years from 2008 onward, following the crisis, and it has not reverted back. The UK authorities did satisfy the traditional rule threshold until 1981, from 1986 to 1990 and from 1997 to 2001. But they violated it from 1990 to 1996 and from 2001 to 2014. Nevertheless, despite these outcomes, the United Kingdom has achieved debt levels below the controlled debt path for more than forty years.

There are a variety of reasons why the traditional rule might have failed. First, the primary surplus may not be able to adjust indefinitely without political or dis-

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model to analyse the behaviour of German and US states, finding that if fiscal transfers are not included in the primary surplus, the US and German state governments pursued unsustainable fiscal policies.
tortionary costs, or because it demands changes that are beyond the economy’s fiscal capacity. Second, it takes time for spending and tax changes to be debated and take effect. If there is any delay in interest rates rising, or if inflation expectations run ahead of actual inflation, the rule will generate a lower primary surplus threshold than is warranted—leading policymakers to assume they have more fiscal space than they actually have. Policymakers will then take on more debt than they should and the ability to stabilise debt is lost. Third, it is a mistake to assume that tax and spending changes are infinitely elastic, if for no other reason there is persistence in entitlement spending and tax changes take time to take effect. All three failures are consequences of having derived the traditional rule from repeated applications of the government’s current budget constraint instead from a consistent application of their inter-temporal budget constraint.

3. An inter-temporal framework for choosing debt and taxation

So what is the best way out of this difficulty; that is, to produce a rule that accounts for the forward looking variables, predictable debt dynamics and the possibility of inter-temporal trade-offs?

What the traditional rules above all leave out is the impact of rising entitlement spending on debt, and how to control that spending. To illustrate how to proceed, this section develops the equivalent debt control rule for that case. Given entitlement spending, predictable over a few periods, this obviously allows for a trade-off between "tax now" policies versus "tax later" policies (i.e. whether to use new debt to finance current spending; or to use tax financing to cover that spending). But the dynamics of debt accumulation must also be taken into account. So the analysis necessarily becomes more complicated and technical. But the final result is only slightly more complicated than the rule we had before, yet is one easily calculated by computer.

To start, we derive an inter-temporal decision framework for a government that
internalises any expected costs arising from raising revenues and issuing taxes, under the assumption that a significant share of expenditure consists of entitlement spending that may be treated as an exogenous stochastic process for the foreseeable future. Our approach is based on an explicit recognition that policy discussions of government spending are generally carried out on the basis that there is a clear-cut difference between discretionary spending, and entitlement spending where funding is mandatory unless specific reforms are proposed in Parliament. Entitlement spending therefore includes all those categories of expenditure that do not have to be approved annually since existing laws give agents rights to certain benefits or services until reforms are called for. Two examples are social security (or social benefits), and the compensation of employees. By contrast, discretionary spending is composed of those expenditures that politicians have to vote on an annual or ad hoc basis in order for them to take place. Discretionary spending also includes categories of intermediate consumption that need to be voted annually in budgetary sessions even if they then recur for several years. Accordingly, we assume that public expenditure \( G_t \) is composed of entitlement spending \( E_t \) and discretionary spending \( V_t \), where entitlement spending is an exogenous stochastic process

\[
G_t = E_t + V_t. \tag{8}
\]

The primary deficit \( D_t \) is the difference between expenditure \( G_t \) and tax revenues \( T_t \) – where total expenditures are total general government expenditures net of interest payments, while taxes include all general government revenues:

\[
D_t = G_t - T_t = -S_t. \tag{9}
\]

By substituting the deficit from Equation (9) into the law of motion of debt, Eq. (6), we get taxes equal to (in each period):

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6 The share of discretionary spending is typically about 1/3 of total government spending in the OECD. A figure this size is relatively large when compared to the estimated residuals of the auto-regressive processes usually used to measure the share of expenditures not predetermined. However, it is smaller than assumed in econometric models that treat government spending as fully exogenous [Coricelli and Fiorito, 2013].

7 All variables are defined as ratios to GDP.
\[ T_t = G_t - B_t + (1 + r_{t-1} - \rho_{t-1})B_{t-1}. \]  

(10)

To keep the model tractable we assume that discretionary spending is financed with tax revenues (but borrowing is allowed to cover entitlement spending):

\[ V_t = \tau T_t \]  

(11)

and thus \( T_t = \psi[E_t - B_t + (1+r_{t-1} - \rho_{t-1}) B_{t-1}] \)

(12)

where \( \tau/(1-\tau) = \psi \) and \( 0 < \tau < 1 \). Since discretionary spending is smaller than entitlement spending, this assumption does not impose any restrictions. The choice of funding by taxation or debt is now made for a given or predicted composition of discretionary and entitlement spending.

Although Eq. 12 is standard, this formulation highlights that, as long as the costs imposed by raising taxes are not linear, any convex cost on \( T_t \) generates an adjustment cost on the stock of debt. Together, these costs impose a ceiling on debt. Eq. 12 can now be rewritten as

\[ T_t = \psi[E_t - \Delta B_t + (r_{t-1} - \rho_{t-1})B_{t-1}] \]  

(13)

As a consequence, whenever linear or non-linear cost functions for taxes apply, the problem becomes dynamic even without assuming non-linear cost functions for the stock of debt or its adjustment.

Our modeling strategy is therefore different from the standard versions which analyse the stock public debt as a sequence of period by period independent budget constraints, without recognising the inter-temporal constraints. In our approach, the objective function of policymakers is to provide the chosen expenditures while minimising taxes and interest costs:

\[ GOF = E_t + \zeta V_t + \frac{\delta}{2}V_t^2 - vT_t - \frac{\phi}{2}T_t^2 - \frac{\iota}{2}B_t^2 - r_tB_t \]  

(14)

Hence entitlement spending generates linear benefits that we normalise to one,

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8 The convexity assumption is supported by the empirical results of Agell (1996) and Agell et al (1996) suggesting that the distortionary effects of taxation are small for low levels of taxation, but they grow rapidly as taxes increase.

9 The constraint we impose is dynamic because it includes both flows and stocks.
and discretionary spending generates both linear and convex benefits whose size is defined by the $\zeta$ and $\delta$ parameters. Similarly, $\nu$ and $\phi$ define linear and non-linear cost parameters associated with tax revenues. They include both resource costs induced by distortionary taxation, and political economy restrictions on government behaviour induced by the voters’ aversion to taxes. Finally, the servicing of debt imposes a standard interest cost $r_t$. But the government needs also to internalise any non-linear cost, $\beta$, caused by non-linearly increasing risk premia on the interest cost, or by the voters’ aversion to public debt. Importantly, since we assume a finite value of $\beta$, we do not consider the case where debt issuance becomes impossible.

Policymakers now maximise (14), subject to (12) and equations that define the stochastic process of expenditure, interest rates, and GDP growth. For example, we might model the latter as exogenous AR(1) processes:

$$E_{t+1} = E_t + \alpha + \eta_{t+1}$$  \hspace{1cm} (15)  
$$r_{t+1} = r_t + \beta + \epsilon_{t+1}$$  \hspace{1cm} (16)  
$$\rho_{t+1} = \rho_t + \gamma + \theta_{t+1}$$  \hspace{1cm} (17)  

where $\eta_{t+1}$, $E_{t+1}$, and $\theta_{t+1}$, represent i.i.d. random shocks with zero mean and constant variances. Defining $d$ as the discount factor, this leads to the Lagrangian form of the problem:

$$\Lambda_t = d^t\{E_t - \gamma T_t - \frac{\alpha}{2} T_t^2 - \frac{\epsilon}{2} B_t^2 - r_t B_t + \mu_t [T_t - \psi(E_t - B_t) - \psi(1 + r_{t-1} - \rho_{t-1} B_{t-1})]$$  \hspace{1cm} (18)  

where $\gamma = \nu - \zeta \tau$ and $\alpha = \phi - \delta \tau^2$. The term in $\alpha$ represents the non-linear cost of taxes, net of benefits, obtained from discretionary spending. Similarly, $\gamma$ measures the linear cost of taxes net of benefits from discretionary spending. Tax revenues are a control variable, while debt is a state variable. The first order conditions for this problem are as follows:

$$\frac{\partial \Lambda}{\partial r_t} = -\gamma - \alpha + \mu_t = 0$$  \hspace{1cm} (19)  
$$\frac{\partial \Lambda}{\partial B_t} = -\mu_t B_t - \psi \mu_t - d\psi(1 + r_t - \rho_t) \mu_{t+1}$$  \hspace{1cm} (20)
\[ \mu_t = \gamma + \alpha T_t \]  

(21)

Now, following Cerniglia et al (2018), define \( \beta = \iota (1 - \tau) \) and \( \nu = 1 - \tau \)\(^{10}\) to get

\[ d(1 + r_t - \rho_t)\mu_{t+1} = \mu_t - \frac{1}{\psi} B_t - \frac{1}{\psi} r_t = \mu_t - \beta B_t - \nu r_t \]  

(22)

Next, eliminating \( \mu_t \) between Eq. 21 and Eq. 22, we obtain:

\[ d(1 + r_t - \rho_t)T_{t+1} = \frac{1}{d} T_t - \frac{1}{d\alpha} [-\gamma (1 - d) + (\nu + d\gamma) r_t - d\gamma \rho_t + \beta B_t] \]  

(23)

Finally, we can now substitute for the value of \( T_t \) in Eq. 9 to obtain:

\[ (1 + r_t - \rho_t)(G_{t+1} - D_{t+1}) = \frac{1}{d} (G_t - D_t) - \frac{1}{d\alpha} [-\gamma (1 - d) + (\nu + d\gamma) r_t - d\gamma \rho_t + \beta B_t] \]  

(24)

and

\[ \Delta B_{t+1} = (1 + r_t - \rho_t)G_{t+1} - \frac{1}{d} G_t - (r_t - \rho_t)B_{t+1} + [(r_t - \rho_t)^2 + 2(r_t - \rho_t)]B_t + \]

\[ \frac{1}{d} B_t - \frac{1}{d} (1 + r_{t-1} - \rho_{t-1})B_t + \frac{1}{d\alpha} [-\gamma (1 - d) + (\nu + d\gamma) r_t - d\gamma \rho_t + \beta B_t] \]  

(25)

At this point, we have derived a law of motion for government debt showing how expenditure shocks are entirely absorbed by new debt issuance. Note: the responses of new issuances to the existing stock of bonds and to market interest rates are both functions of the relative weights of the non-linear cost of debt and taxes. For higher levels of the tax parameter \( \alpha \), the impact of interest rates and the past stock of debt declines. The impact of interest rates rises with the linear cost term \( \gamma \), while the impact of the past stock of debt is large for larger values of \( \beta \) reflecting the convex costs of the stock of debt.

This second order difference equation in \( B_{t+1} \) (which includes the debt implicit in the anticipated deficits \( D_{t+1} \) and current deficit \( D_t \)) cannot be solved with the standard techniques proposed by Sargent (1979) because \( r_t \) is part of the intercept term and also of the slope. The roots in the solutions are thus time varying.

\[^{10}\] Thus \( \iota < \beta \) and \( \nu < 1 \). In practice the parameters of this model are likely to vary over time and different economies.
5. The new solvency rule

Finally, the law of motion derived in Equation (25) can be rewritten to make the role of primary deficits and surpluses explicit:

\[
\Delta B_{t+1} = (1 + r_t - \rho_t)G_{t+1} + \frac{1}{d} G_t + (r_t - \rho_t)(1 + r_{t-1} - \rho_{t-1})B_{t-1} -
\]

\[
(r_t - \rho_{t-1})\Delta D_t + \frac{1}{\alpha} D_t + \frac{1}{d\alpha} [-\gamma(1 - d) + (\nu + d\gamma)r_t - d\gamma\rho_t + \beta B_t]
\]

which yields a rule that defines a primary surplus sufficient to stabilise debt:

\[
-D_t = [d(1 + r_t - \rho_t)G_{t+1} - G_t] + d(r_t - \rho_t)(B_t - D_{t+1}) -
\]

\[
\frac{1}{\alpha} [-\gamma(1 - d) + (\nu + d\gamma)r_t - d\gamma\rho_t + \beta B_t]
\]

where \(\nu = 1 - \tau\). This rule defines a threshold above which the primary surplus will reduce the debt ratio, and below which debt increases. It is however, quite different from the standard textbook rule which would be \(-D_t = (r_t - \rho_t)B_t\) in this notation.

In fact (27) contains three terms. a) The first represents the discounted growth adjusted increase in total public spending relative to national income. b) The second is the counterpart to the standard textbook rule, but discounted and adjusted for the expected/planned surplus next period. c) The last term represents all the preference factors (and post-tax interest rate) that arise because we need to account for inter-temporal optimising behaviour: that is, for the choice between raising taxes now; or to extend debt to tax later to finance current or future spending liabilities. The trade-off between using taxes vs. debt is likely to dominate (27) since the remaining terms will be small compared to \((\beta/\alpha)B_t\).

The practical implication of (27) is that the old period-by-period criterion is no longer sufficient to ensure sustainable public finances once entitlement spending enters the story. Instead, we need to impose a condition that public spending shall not grow faster than national income by a certain margin and then subtract off the effect of preferring to tax now vs. increasing debt and tax later using expected growth to compensate. But none of these choices can be made without cost or restriction. When policymakers have a strong aversion to debt, so that \(\beta\) is larger, or when
interest costs are high, the primary surplus required is larger. By contrast, the higher the distortions caused by high tax revenues relative to income, the lower is the primary surplus required since debt becomes a more attractive tool to smooth out any expenditure shocks.

5. Conclusions

This paper has introduced a primary surplus rule to ensure stable and sustainable public finances in a dynamic world with entitlement spending liabilities. To do that requires us to move from a static rule based on the government’s current budget constraint, to an inter-temporal rule, since policymakers have to trade off using taxes now to fund current (future) spending vs. using debt, and future taxes, to fund that spending. Our rule therefore replaces the traditional period-by-period rule, and responds to three different sets of variables: a) the growth of public spending relative to the growth in national income; b) the familiar growth adjusted interest payments due on current debt, now discounted and adjusted for anticipated future primary surpluses; and c) the effects of choosing between taxing now, or later by increasing current debt.

Thus, in a dynamic world, the primary surplus needs to at least match any expected discounted increases in public spending, the net interest on existing debt, and terms reflecting the preference (or cost) for extending debt relative to the cost of changing taxes. When policymakers have a strong aversion to debt (β is large), or if interest costs relative to growth are high, the primary surplus required is larger. But if the distortions due to high taxes relative to income are large, a lower primary surplus is required since debt becomes a more attractive tool to smooth spending shocks.

It is important to note that some countries have capped spending growth to be no more than national income growth, alongside a standard primary surplus rule or a balanced budget rule. A prominent example is the Euro-zone’s fiscal compact. But none have accounted for the impact of preferences when spending is dynamic; or for the projected impact of interest payments on future deficits; or have considered
limiting spending growth to the available space, instead of a hard limit to GDP growth which would institutionalise austerity policies in every recession. Thus, none of those operating rules are sufficient to guarantee fiscal sustainability. But they might be simple and possibly effective approximations to our more complicated rule. It remains to be determined if (and when) that is true.

**References**


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