Supplementary Material to *"Nowcasting from Cross-Sectionally Dependent Panels"*

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Abstract

This Supplementary Material contains a number of additional details and empirical results from paper *"Nowcasting from Cross-Sectionally Dependent Panels."* Specifically, we start out with more step-by-step details on how we derive our proposed panel nowcasting method from the paper of Chudik & Pesaran (2015). Then we present empirical findings not included in the main text for the GDP example: firstly some additional robustness with regards to additional predictor variables and then robustness to the out-of-sample split ratio.

JEL Classification: C13, C23, C33, C53
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S1 Details on Model Estimation and Derivation

In the main paper we present the full mixed-frequency PMIDAS nowcasting model with a ragged edge. In this section, we go back and construct the PMIDAS model step-by-step in two phases, starting from the dynamic CSD panel data model of Chudik & Pesaran (2015). As in the main paper, we have a target variable of interest $y_{i,t}$ for the i^{th} cross-sectional unit and t^{th} time point, where $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$. Let $x_{i,t}$ be a predictor which, for the sake of simplicity in this Supplementary Material, we assume is a single variable. This is easily generalised to the case of many predictors as in the main text. In this first step we consider $x_{i,t}$ and $y_{i,t}$ to be of the same frequency. This allows us to directly modify the CCE estimation method of Chudik & Pesaran (2015) to the lagged version, i.e. LCCE, which permits the model to be used for forecasting. In the second step, the model is further extended to incorporate the mixed frequency data as in general nowcasting frameworks. Here the predictor variable $x_{i,t}$ is assumed to be of a higher frequency than that of the target $y_{i,t}$. Finally we add in the ragged edge where lag structures and model parameters depend on the nowcast day, v.

S1.1 The Nowcasting Model: Single Frequency, no Ragged Edge

S1.1.1 Set-up

The main framework for the dynamic heterogeneous panel model with multi-factor error structure follows the format of Chudik & Pesaran (2015):

$$y_{i,t} = c_i + \phi_i y_{i,t-1} + \beta_{0i} x_{i,t} + \beta_{1i} x_{i,t-1} + u_{i,t} \tag{S1a}$$
$$u_{i,t} = \gamma_i' f_t + \varepsilon_{i,t} \tag{S1b}$$
$$x_{i,t} = \kappa_i + \alpha_i y_{i,t-1} + \Gamma_i' f_t + \epsilon_{i,t} \tag{S1c}$$

In the above equations, c_i and κ_i are individual fixed effects. The term f_t is an $m \times 1$ vector of unobserved common factors which impact both the target and the predictor through loadings γ_i and Γ_i which are of orders $m \times 1$ respectively. The coefficient α_i characterises the relation between the predictor and lagged target variables, $\varepsilon_{i,t}$ represents the idiosyncratic errors and $\epsilon_{i,t}$ is assumed to follow a general linear covariance stationary process distributed independently of $\varepsilon_{i,t}$.

In the original formulation of the CCE approach, the common factors, f_t, are estimated using the cross-sectional averages of $z_{i,t} = (y_{i,t}, x_{i,t})'$. The presence of $y_{i,t}$ in the estimates of the factors clearly makes the model unsuitable for forecasting or nowcasting applications. Therefore, we propose the following modifications.

S1.1.2 Estimation and Nowcasting

We define $z_{i,t}$ to contain lagged y and the current information on the predictor variable x, i.e.,

$$z_{i,t} = \begin{pmatrix} y_{i,t-1} \\ x_{i,t} \end{pmatrix}$$

Combining this with equations S1a, S1b and S1c we obtain:

$$A_{0i} z_{i,t} = c_{zi} + A_{1i} z_{i,t-1} + A_{2i} z_{i,t-2} + C_i F_t + e_{i,t} \tag{S2a}$$
$$\implies z_{i,t} = K_{zi} + B_{0i} z_{i,t-1} + B_{1i} z_{i,t-2} + A_{0i}^{-1} C_i F_t + A_{0i}^{-1} e_{i,t} \tag{S2b}$$

where:

$$
\begin{aligned}
&c_{zi} = \begin{pmatrix} c_i \\ \kappa_i \end{pmatrix}, A_{0i} = \begin{pmatrix} 1 & 0 \\ -\alpha_i & 1 \end{pmatrix}, A_{1i} = \begin{pmatrix} \phi_i & \beta_{0i} \\ 0 & 0 \end{pmatrix}, A_{2i} = \begin{pmatrix} 0 & \beta_{1i} \\ 0 & 0 \end{pmatrix} \\
&K_{zi} = A_{0i}^{-1} c_{zi},\ B_{0i} = A_{0i}^{-1} A_{1i},\ B_{1i} = A_{0i}^{-1} A_{2i} \\
&C_i = \begin{pmatrix} 0 & \gamma_i' \\ \Gamma_i' & 0 \end{pmatrix} = \begin{pmatrix} C_{0i} & C_{1i} \end{pmatrix}\ C_{0i} = \begin{pmatrix} 0 \\ \Gamma_i' \end{pmatrix}\ C_{1i} = \begin{pmatrix} \gamma_i' \\ 0 \end{pmatrix}, F_t = \begin{pmatrix} f_t \\ f_{t-1} \end{pmatrix} \text{ and } e_{i,t} = \begin{pmatrix} \varepsilon_{i,t-1} \\ \epsilon_{i,t} \end{pmatrix}
\end{aligned}
$$

Assumption 1. *The eigenvalues of the following augmented matrix are less than one in absolute value:*

$$
\begin{pmatrix} B_{0i} & B_{1i} \\ I & 0 \end{pmatrix}
$$

Next, we derive the large-N representation of the factors, f_t, in terms of the cross-sectional averages of $z_{i,t}$. With assumption 1, $z_{i,t}$ is an invertible covariance stationary process and can be written as:

$$
\begin{aligned}
(I - B_{0i}L - B_{1i}L^2) z_{i,t} &= K_{zi} + A_{0i}^{-1} C_i F_t + A_{0i}^{-1} e_{i,t} && \text{(S3a)} \\
\implies z_{i,t} &= K_{1_{zi}} + \Psi_i(L) A_{0i}^{-1} C_i F_t + \Psi_i(L) A_{0i}^{-1} e_{i,t} && \text{(S3b)} \\
\implies z_{i,t} - K_{1_{zi}} &= \Psi_i(L) A_{0i}^{-1} C_i F_t + \Psi_i(L) A_{0i}^{-1} e_{i,t} && \text{(S3c)}
\end{aligned}
$$

where:

$$
\begin{aligned}
K_{1_{zi}} &= (I - B_{0i}L - B_{1i}L^2)^{-1} K_{zi} \\
\Psi_{i0} &= I, \Psi_{i1} = B_{0i} \\
\Psi_{iv} &= B_{0i}\Psi_{i,v-1} + B_{1i}\Psi_{i,v-2}, v \geq 2
\end{aligned}
$$

The assumptions 3-5 of Chudik & Pesaran (2015) ensure that the $\Psi_i(L)$ coefficient matrices are independently distributed of each other and also over the cross-sections. We take weighted cross-sectional averages of equation S3c, using a weight vector $w = (\omega_1, \omega_2, ..., \omega_N)'$. Assuming the granularity conditions and using similar steps as in Chudik & Pesaran (2015), the terms on the RHS of equation S3c give:

$$
\begin{aligned}
\sum_{i=1}^{N} \left[\sum_{l=0}^{\infty} \omega_i \Psi_{il} A_{0i}^{-1} C_i F_{t-l} \right] &= \sum_{l=0}^{\infty} E\left[\Psi_{il} A_{0i}^{-1} C_i \right] F_{t-l} + O_p(N^{-\frac{1}{2}}) \\
&= \Lambda(L) C F_t + O_p(N^{-\frac{1}{2}}) && \text{(S4a)} \\
\text{and } \sum_{i=1}^{N} \left[\sum_{l=0}^{\infty} \omega_i \Psi_i(L) A_{0i}^{-1} e_{i,t} \right] &= O_p(N^{-\frac{1}{2}}) && \text{(S4b)}
\end{aligned}
$$

where:

$$\Lambda(L) = \sum_{l=0}^{\infty} \Lambda_l L^l = \sum_{l=0}^{\infty} E\left[\Psi_{il} A_{0i}^{-1}\right] L^l$$

$$C = E[C_i] = E\begin{pmatrix} 0 & \gamma_i' \\ \Gamma_i' & 0 \end{pmatrix} = \begin{pmatrix} C_0 & C_1 \end{pmatrix}$$

$$E(C_{0i}) = C_0 \text{ and } E(C_{1i}) = C_1$$

Assumption 2. *The inverse of the matrix* $\Lambda(L) = \sum_{l=0}^{\infty} \Lambda_l L^l = \sum_{l=0}^{\infty} E\left[\Psi_{il} A_{0i}^{-1}\right] L^l$ *exists and has exponentially decaying coefficients.*

Continuing from S4a we have:

$$\sum_{i=1}^{N}\left[\sum_{l=0}^{\infty} \omega_i \Psi_{il} A_{0i}^{-1} C_i F_{t-l}\right] = \Lambda(L)\left[C_0 + C_1 L\right] f_t + O_p(N^{-\frac{1}{2}}) \tag{S5}$$

Defining the detrended weighted cross-sectional averages from the LHS of equation S3c as $\widetilde{z}_{wt} = \sum_{i=1}^{N} \omega_i (z_{i,t} - K_{1_{zi}}) = \sum_{i=1}^{N} \omega_i z_{i,t} - \overline{c}_{zw}$, where $\overline{c}_{zw} = \sum_{i=1}^{N} \omega_i K_{1_{zi}}$, we obtain the following large-N representation of the detrended cross-sectional averages $\widetilde{z}_{wt}$:

$$\widetilde{z}_{wt} = \Lambda(L)\left[C_0 + C_1 L\right] f_t + O_p(N^{-\frac{1}{2}}) \tag{S6}$$

$$\implies \Lambda^{-1}(L)\widetilde{z}_{wt} = \left[C_0 + C_1 L\right] f_t + O_p(N^{-\frac{1}{2}}) \tag{S7}$$

Assumption 3. C_0 *has full column rank.*

Assumption 4. *The eigenvalues of* $(C_0' C_0)^{-1} C_0' C_1$ *are less than unity in absolute value.*

Assumptions 3 and 4 are crucial for the estimation of the unit-specific coefficients. From equation S6 we have:

$$f_t = G(L)\widetilde{z}_{wt} + O_p(N^{-\frac{1}{2}}) \tag{S8}$$

where:

$$G(L) = \left[I + (C_0' C_0)^{-1} C_0' C_1 L\right]^{-1} \left[C_0' C_0\right]^{-1} C_0' \Lambda^{-1}(L)$$

Substituting the large-N representation of the unobserved common factors from equation S8 into equation S1a, in a similar way to Chudik & Pesaran (2015) we obtain an expression for $y_{i,t}$ as a function of the cross-sectional weighted averages $\overline{z}_{wt} = \sum_{i=1}^{N} \omega_i z_{i,t}$ as follows:

$$y_{i,t} = c_i^* + \phi_i y_{i,t-1} + \beta_{0i} x_{i,t} + \beta_{1i} x_{i,t-1} + \delta_i'(L)\overline{z}_{wt} + \varepsilon_{i,t} + O_p(N^{-\frac{1}{2}}) \tag{S9a}$$

$$= c_i^* + \phi_i y_{i,t-1} + \beta_{0i} x_{i,t} + \beta_{1i} x_{i,t-1} + \sum_{l=0}^{p_T} \delta_{il}' \overline{z}_{w,t-l} + e_{i,t} \tag{S9b}$$

where:

$$e_{i,t} = \varepsilon_{i,t} + \sum_{l=p_T+1}^{\infty} \delta_{il}' \overline{z}_{w,t-l} + O_p(N^{-\frac{1}{2}})$$

$$c_i^* = c_i - \delta_i'(1)\overline{c}_{zw}$$

and:

$$\delta_i(L) = \sum_{l=0}^{\infty} \delta_{il} L^l = \gamma_i' G(L) \tag{S9c}$$

and p_T is the truncation used for the infinite lag polynomial of equation S9c.

The nowcasting approach is now based on least squares estimation of equation S9b, under assumptions 1-4 and 7 of Chudik & Pesaran (2015), in addition to the ones stated above. As in the main text, if one wishes to shut down parameter heterogeneity, the model can be estimated by pooled OLS. These steps show how the contemporaneous CCE estimation of the CSD panel models can be modified to LCCE which uses the lagged target variable and henceforth can be used for forecasting or nowcasting applications.

S1.2 The Nowcasting Model: Mixed-Frequency, no Ragged Edge

S1.2.1 Set-up

To extend the above nowcasting framework to include mixed frequency data (though still no ragged edge, so no dependence on v), consider the single predictor variable to be of a higher frequency relative to the target variable. As in the main text, we take the example where $y_{i,t}$ is of quarterly frequency, and let the predictor variable be monthly and denoted by $x^M_{i,t}$. The ratio of frequencies can be easily generalised. We use the stacked high-frequency process $X^M_{i,t}$:

$$X^M_{i,t} = \begin{pmatrix} x^M_{i,t} \\ x^M_{i,t-\frac{1}{3}} \\ x^M_{i,t-\frac{2}{3}} \end{pmatrix}$$

which was defined in the main text in equation 1.

To extend the panel model to mixed frequency, the lags of the high-frequency process are directly included in equations S1a, S1b and S1c in line with UMIDAS-type models of Foroni et al. (2015) and others. Hence, we adapt the previous model to get the following mixed-frequency dynamic heterogeneous panel data model with multi-factor error structure:

$$y_{i,t} = c_i + \phi_i y_{i,t-1} + \beta_{0i} x^M_{i,t} + \beta_{1i} x^M_{i,t-\frac{1}{3}} + \beta_{2i} x^M_{i,t-\frac{2}{3}} + u_{i,t} \tag{S10a}$$

$$u_{i,t} = \gamma_i' f_t + \varepsilon_{i,t} \tag{S10b}$$

$$X^M_{i,t} = \kappa_i + \alpha_i y_{i,t-1} + \Gamma_i' f_t + \epsilon_{i,t} \tag{S10c}$$

where we adopt the same notation as in the previous section for simplicity, noting that the parameters κ_i and α_i and the errors $\epsilon_{i,t}$ are now vectors and Γ_i is a matrix, in order to match the dimension of $X^M_{i,t}$ in the mixed frequency set-up.

Equation S10a is the panel equivalent of a UMIDAS model with no functional distributed lag polynomials. Foroni et al. (2015) conclude that UMIDAS performs better as compared to other functional lag MIDAS in case the differences in frequencies is not too high, particularly in the quarterly to monthly frequency mix, as in the empirical application later. This also suits the linear estimation framework of LCCE described above as UMIDAS models, unlike other MIDAS specifications, do not have to be estimated by non-linear least squares.

The entire system of equations can be cast into an MFVAR representation constructed using stacked skip-sampled processes (Ghysels 2016). In our case, the MFVAR is already in a reduced form, with restricted parameter space as in Ghysels (2018). Additionally, the MFVAR here is extended to the case of panel data (the MF-PVAR) and with a multi-factor error structure. To see this explicitly, construct the stacked compact expression of the equations S10a, S10b and S10c as below:

$$h_{i,t} = \begin{pmatrix} y_{i,t} \\ X^M_{i,t} \end{pmatrix}$$

$$K_{0i} h_{i,t} = c_i + K_{1i} h_{i,t-1} + C_i f_t + e_{i,t} \tag{S11}$$

Equation S11 gives the panel extension of the reduced form MIDAS-VAR model. where:

$$K_{0i} = \begin{pmatrix} 1 & -\beta_{0i} & -\beta_{1i} & -\beta_{2i} \\ 0 & & I & \end{pmatrix}, \; c_i = \begin{pmatrix} c_i \\ \kappa_i \end{pmatrix}, \; K_{1i} = \begin{pmatrix} \phi_i & 0 \\ \alpha_i & 0 \end{pmatrix}$$

$$C_i = \begin{pmatrix} \gamma_i' \\ \Gamma_i' \end{pmatrix}, \; e_{i,t} = \begin{pmatrix} \varepsilon_{i,t} \\ \epsilon_{i,t} \end{pmatrix}$$

S1.2.2 Estimation and Nowcasting

To estimate the factors in the mixed frequency set-up, the process, in essence, remains quite similar to that in subsection S1.1.2. We redefine the stacked vector, $z^M_{i,t}$, of the lagged target variable and the stacked predictor variable, as well as the β parameters, as follows:

$$z^M_{i,t} = \begin{pmatrix} y_{i,t-1} \\ X^M_{i,t} \end{pmatrix} \quad \beta_i = \begin{pmatrix} \beta_{0i} \\ \beta_{1i} \\ \beta_{2i} \end{pmatrix}$$

Lagging equations S10a, and S10b and writing the system in a stacked compact matrix notation:

$$\begin{aligned} \begin{pmatrix} 1 & 0 \\ -\alpha_{xi} & I \end{pmatrix} \begin{pmatrix} y_{i,t-1} \\ X^M_{i,t} \end{pmatrix} &= \begin{pmatrix} c_i \\ \kappa_i \end{pmatrix} + \begin{pmatrix} \phi_i & \beta_i' \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{i,t-2} \\ X^M_{i,t-1} \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 & \gamma_i' \\ \Gamma_i' & 0 \end{pmatrix} \begin{pmatrix} f_t \\ f_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,t-1} \\ \epsilon_{i,t} \end{pmatrix} \end{aligned} \tag{S12}$$

This gives the reduced form MFVAR expression and rest of the estimation process can now be carried out as described in S1.1.2 with the stacked skip-sampled high-frequency predictor variable $X^M_{i,t}$. The final mixed frequency panel nowcasting equations are:

$$y_{i,t} = c_i^* + \phi_i y_{i,t-1} + \beta_i' X^M_{i,t} + \delta_i'(L) \overline{z}^M_{wt} + \varepsilon_{i,t} + O_p(N^{-\frac{1}{2}}) \tag{S13a}$$

$$= c_i^* + \phi_i y_{i,t-1} + \beta_i' X^M_{i,t} + \sum_{l=0}^{p_T} \delta_{il}' \overline{z}^M_{t-l} + e_{i,t} \tag{S13b}$$

where $\overline{z}^M_t = \sum_{i=1}^N \omega_i z^M_{i,t}$ is the equivalent cross-sectionally weighted average as in the previous section, this time modified for the mixed-frequency set-up. We note that, as in Chudik & Pesaran (2015), an additional set of variables (for instance $g_{i,t}$) may be used in the cross-sectional averages to estimate the factors. The idea here is that the variables $g_{i,t}$ are also impacted by the same common factors. This is quite common in macroeconomic databases, where a handful of factors capture the information contained in large sets of indicators. Thus, the model can be further enriched by the information contained in other high-frequency macro-series, which do not enter the main nowcasting equation.

S1.3 The Nowcasting Model: Mixed-Frequency, Ragged Edge

Finally, we now turn our attention to the main nowcasting approach with mixed frequencies and the ragged edge, as outlined in the main text. As described there, the incorporation of country-level calendar effects requires additional notation:

1. The nowcast is performed on the v^{th} day of the nowcast quarter;
2. m_{iv} : The monthly lag available for the high-frequency variable for the cross-section i on the v^{th} day of the nowcast quarter;[1]
3. d_{iv} : The quarterly lag available for the high-frequency variable for the cross-section i on the v^{th} day of the nowcast quarter.

The PMIDAS model equations (S10a, S10b and S10c) in this set up are then modified as follows (taking again the single variable case, unlike the multiple variable case in the main text), with a lag structure which depends on d_{iv} and m_{iv}, as well as model parameters that depend on v:

$$y_{i,t} = c_{vi} + \phi_{vi} y_{i,t-d_{iv}} + \beta'_{vi} X^M_{i,t-\frac{m_{iv}}{3}} + \gamma'_{vi} f_t + \varepsilon_{v,i,t} \tag{S14a}$$

$$X^M_{i,t-\frac{m_{iv}}{3}} = \kappa_{vi} + \alpha_{vi} y_{i,t-d_{iv}} + \Gamma'_{vi} f_t + \epsilon_{v,i,t} \tag{S14b}$$

where $X^M_{i,t}$, as above, is the stacked vector defined in equation 1 in the main text. Lagging equation S14a by d_{iv} periods, and manipulating equation S14b gives the following:

$$y_{i,t-d_{iv}} = c_{vi} + \phi_{vi} y_{i,t-2d_{iv}} + \beta'_{vi} X^M_{i,t-\frac{m_{iv}-d_{iv}}{3}} + \gamma'_{vi} f_{t-d_{iv}} + \varepsilon_{v,i,t-d_{iv}} \tag{S15a}$$

$$-\alpha_{vi} y_{i,t-d_{iv}} + X^M_{i,t-\frac{m_{iv}}{3}} = \kappa_i + \Gamma'_{vi} f_t + \epsilon_{v,i,t} \tag{S15b}$$

Stacking this into one system yields the following:

$$\begin{aligned}\begin{pmatrix} 1 & 0 \\ -\alpha_{vi} & I \end{pmatrix} \begin{pmatrix} y_{i,t-d_{iv}} \\ X^M_{i,t-\frac{m_{iv}}{3}} \end{pmatrix} = \begin{pmatrix} c_{vi} \\ \kappa_{vi} \end{pmatrix} + \begin{pmatrix} \phi_{vi} & \beta'_{vi} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{i,t-2d_{iv}} \\ X^M_{i,t-\frac{m_{iv}}{3}-d_{iv}} \end{pmatrix} \\ + \begin{pmatrix} 0 & \gamma'_{vi} \\ \Gamma'_{vi} & 0 \end{pmatrix} \begin{pmatrix} f_t \\ f_{t-d_{iv}} \end{pmatrix} + \begin{pmatrix} \varepsilon_{v,i,t-d_{iv}} \\ \epsilon_{v,i,t} \end{pmatrix}\end{aligned} \tag{S16}$$

Finally, we modify the stacked vector from before to get:

$$z^M_{i,t,v} = \begin{pmatrix} y_{i,t-d_{iv}} \\ X^M_{i,t-\frac{m_{iv}}{3}} \end{pmatrix} \tag{S17}$$

[1]Recall that, for simplicity, we use a single predictor variable in the model. With multiple predictors, m_{iv} would also potentially be different across variables.

as in equation 5 of the main text. So the MFVAR is written as:

$$A_{0i} z^M_{i,t,v} = c_{zi} + A_{1i} z^M_{i,t-d_{iv},v} + \left[C_{0i} + C_{1i} L^{d_{iv}}\right] f_t + e_{v,i,t} \tag{S18a}$$

$$\implies z^M_{i,t,v} = K_{zi} + B_{0i} z^M_{i,t-d_{iv},v} + A_{0i}^{-1} \left[C_{0i} + C_{1i} L^{d_{iv}}\right] f_t + A_{0i}^{-1} e_{v,i,t} \tag{S18b}$$

$$\implies (I - B_{0i} L^{d_{iv}}) z^M_{i,t,v} = K_{zi} + A_{0i}^{-1} \left[C_{0i} + C_{1i} L^{d_{iv}}\right] f_t + A_{0i}^{-1} e_{v,i,t} \tag{S18c}$$

where:

$$e_{v,i,t} = \begin{pmatrix} \varepsilon_{v,i,t-d_{iv}} \\ \epsilon_{v,i,t} \end{pmatrix}$$

and the rest of the matrices (A_{0i}, B_{0i} and others, suppressing dependence of these on v to avoid further notational clutter) have the similar definitions as earlier. Further manipulation yields:

$$z^M_{i,t,v} = K_{1_{zi}} + \Psi_i(L^{d_{iv}}) A_{0i}^{-1} \left[C_{0i} + C_{1i} L^{d_{iv}}\right] f_t + \Psi_i(L^{d_{iv}}) A_{0i}^{-1} e_{v,i,t} \tag{S18d}$$

To estimate the factors, we take the weighted cross-sectional averages of the above. The first term of the RHS gives:

$$\sum_{i=1}^{N} \left[\omega_i \Psi_i(L^{d_{iv}}) A_{0i}^{-1} \left\{C_{0i} + C_{1i} L^{d_{iv}}\right\} f_t\right] = \sum_{l=0}^{\infty} \sum_{i=1}^{N} \omega_i \Psi_{il} A_{0i}^{-1} L^{l d_{iv}} \left[C_{0i} + C_{1i} L^{d_{iv}}\right] f_t \tag{S18e}$$

The following generalised assumption replaces the assumptions 2-4 stated earlier to estimate the factors using the cross-sectional averages of $z^M_{i,t,v}$ as defined in equation 5 i.e. f_t can be approximated by a finite number of lags of $\sum_{i=1}^{N} \omega_i z^M_{i,t,v}$.

Assumption 5. *The weighted average of the lag polynomials on the RHS of equation S18e is invertible and the inverse polynomial has exponentially decaying coefficients:*

$$\sum_{l=0}^{\infty} \left[\sum_{i=1}^{N} \omega_i \Psi_{il} A_{0i}^{-1}\right] \left[C_{0i} + C_{1i} L^{d_{iv}}\right] L^{l d_{iv}}$$

Given that d_{iv} typically only takes on a handful of values, with $\{0, 1, 2\}$ being the exhaustive set in the GDP nowcasting example, we may have a weighted average of a maximum of two polynomials in assumption 5. Taking the example of the U.S. and Germany, if we nowcast Q1 on the 2nd of February, d_{iv} takes values 1 and 2 respectively for the US and Germany i.e. $v = 33$, $d_{US,33} = 1$, for the US and $d_{GER,33} = 2$ for Germany. So, in this case, we have a weighted average of two lagged polynomials, which is assumed to be invertible by assumption 5.

S2 Additional Monte Carlo Results - $q = 4$

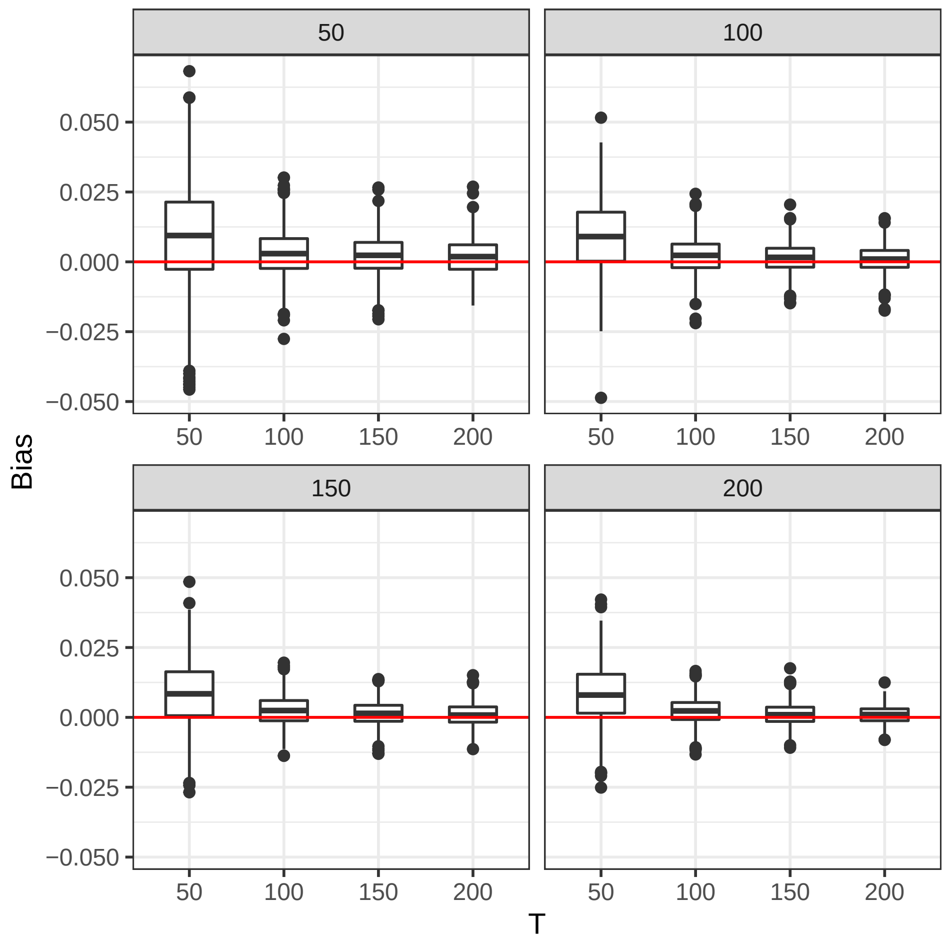
In this section we present the additional Monte Carlo results when the frequency mix is changed from $q = 3$ to $q = 4$ which is relevant for annual-quarterly or monthly-weekly panel nowcasting exercises.

Table S1: Simulation Results - Absolute Bias in LCCE and CCE ($q = 4$)

	CCE				**LCCE**			
N/T	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
				ϕ				
50	0.0627	0.0278	0.0173	0.0133	0.0540	0.0253	0.0162	0.0126
100	0.0635	0.0273	0.0176	0.0128	0.0543	0.0253	0.0165	0.0123
150	0.0636	0.0279	0.0179	0.0126	0.0552	0.0254	0.0166	0.0118
200	0.0647	0.0279	0.0173	0.0126	0.0564	0.0256	0.0163	0.0117
				$\beta^{(0)}$				
50	0.0247	0.0135	0.0105	0.0093	0.0241	0.0146	0.0126	0.0118
100	0.0174	0.0096	0.0076	0.0062	0.0171	0.0107	0.0089	0.0077
150	0.0134	0.0082	0.0060	0.0050	0.0135	0.0088	0.0069	0.0065
200	0.0125	0.0071	0.0052	0.0045	0.0120	0.0077	0.0063	0.0056
				$\beta^{(1)}$				
50	0.0254	0.0141	0.0109	0.0092	0.0240	0.0159	0.0129	0.0118
100	0.0181	0.0103	0.0076	0.0066	0.0173	0.0116	0.0091	0.0081
150	0.0149	0.0085	0.0061	0.0049	0.0145	0.0096	0.0076	0.0068
200	0.0123	0.0070	0.0052	0.0045	0.0119	0.0076	0.0060	0.0055
				$\beta^{(2)}$				
50	0.0259	0.0143	0.0109	0.0090	0.0248	0.0154	0.0129	0.0117
100	0.0180	0.0097	0.0080	0.0066	0.0171	0.0110	0.0093	0.0082
150	0.0146	0.0083	0.0063	0.0053	0.0144	0.0092	0.0074	0.0067
200	0.0127	0.0071	0.0055	0.0044	0.0123	0.0079	0.0067	0.0056
				$\beta^{(3)}$				
50	0.0250	0.0144	0.0110	0.0095	0.0243	0.0157	0.0130	0.0118
100	0.0180	0.0101	0.0075	0.0062	0.0167	0.0108	0.0094	0.0079
150	0.0145	0.0083	0.0065	0.0054	0.0147	0.0091	0.0076	0.0066
200	0.0125	0.0071	0.0055	0.0046	0.0124	0.0079	0.0065	0.0057

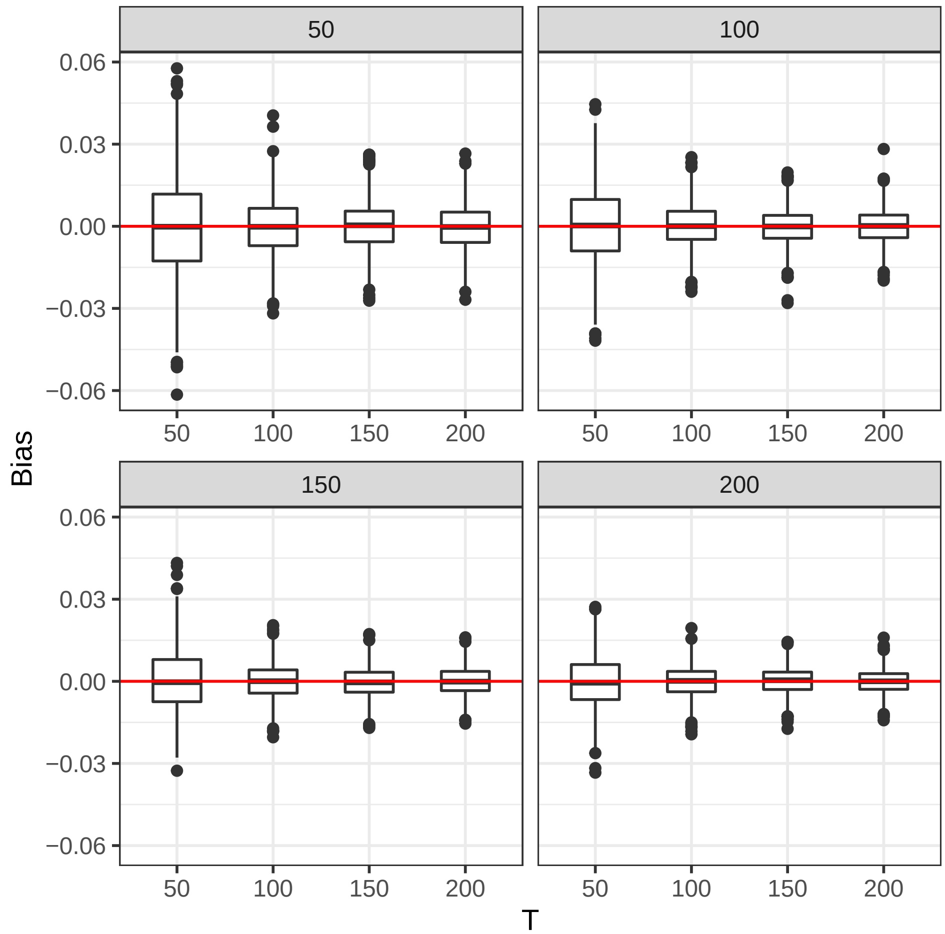
Notes: The numbers in this table are the absolute biases in the estimates of the key model parameters estimated using two methods, LCCE and CCE, across different sample sizes.

Figure S1: Bias in ϕ; $q = 4$



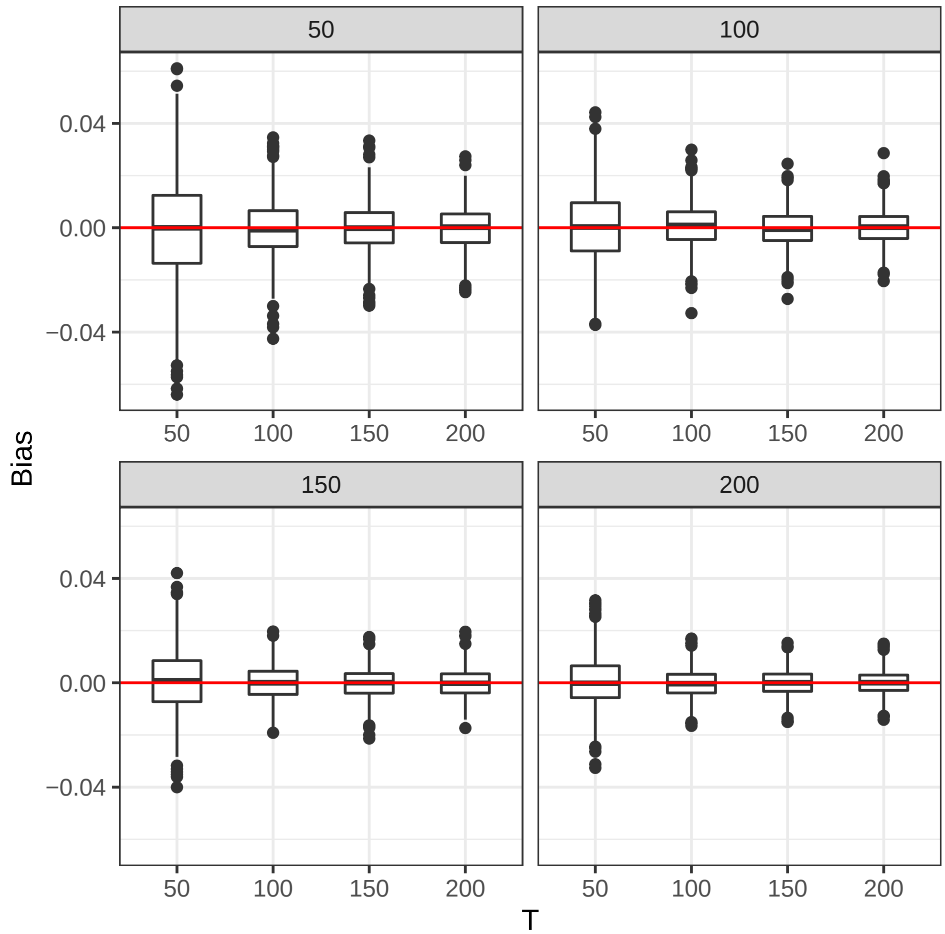
Notes: The figures display the distribution of the difference in absolute biases of the parameter ϕ for the estimation methods LCCE relative to CCE. Figures lower than zero mean that LCCE has lower absolute bias than CCE. The panel header show the number of cross-sections

Figure S2: Bias Comparison in $\beta^{(0)}$; $q = 4$



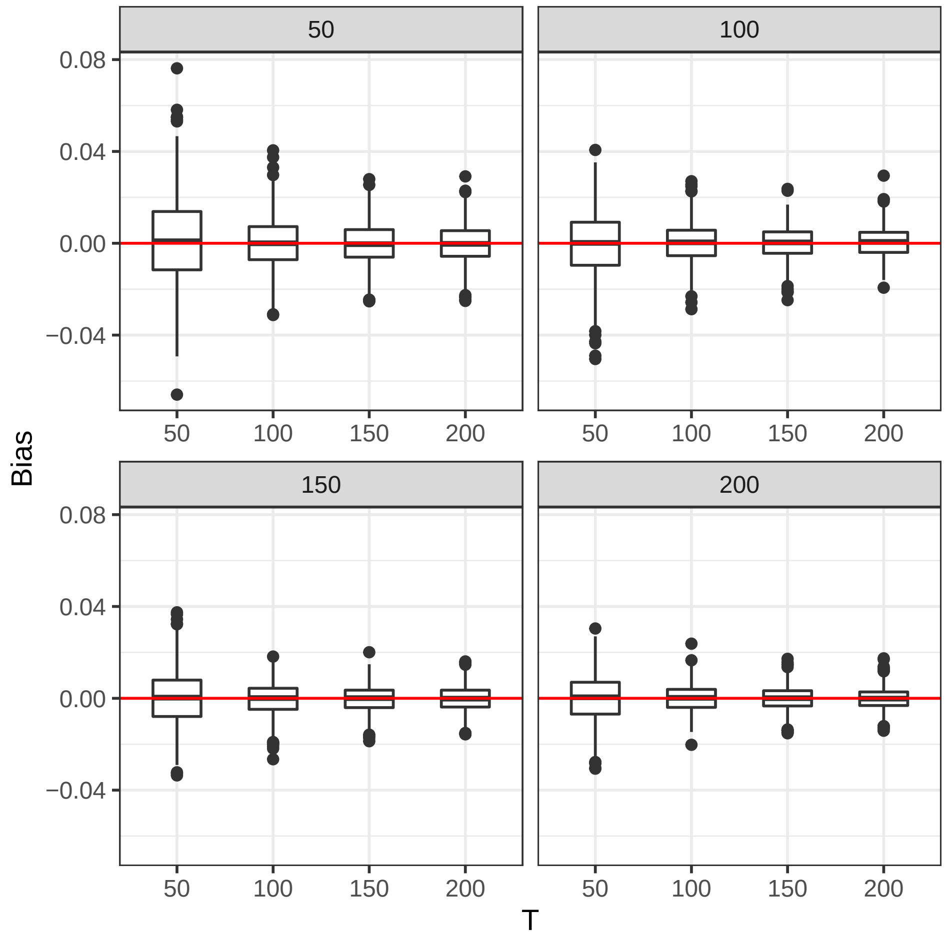
Notes: The figures display the distribution of the difference in absolute biases of the parameter $\beta^{(0)}$ for the estimation methods LCCE relative to CCE. Figures lower than zero mean that LCCE has lower absolute bias than CCE. The panel header show the number of cross-sections

Figure S3: Bias Comparison in $\beta^{(1)}$; $q = 3$



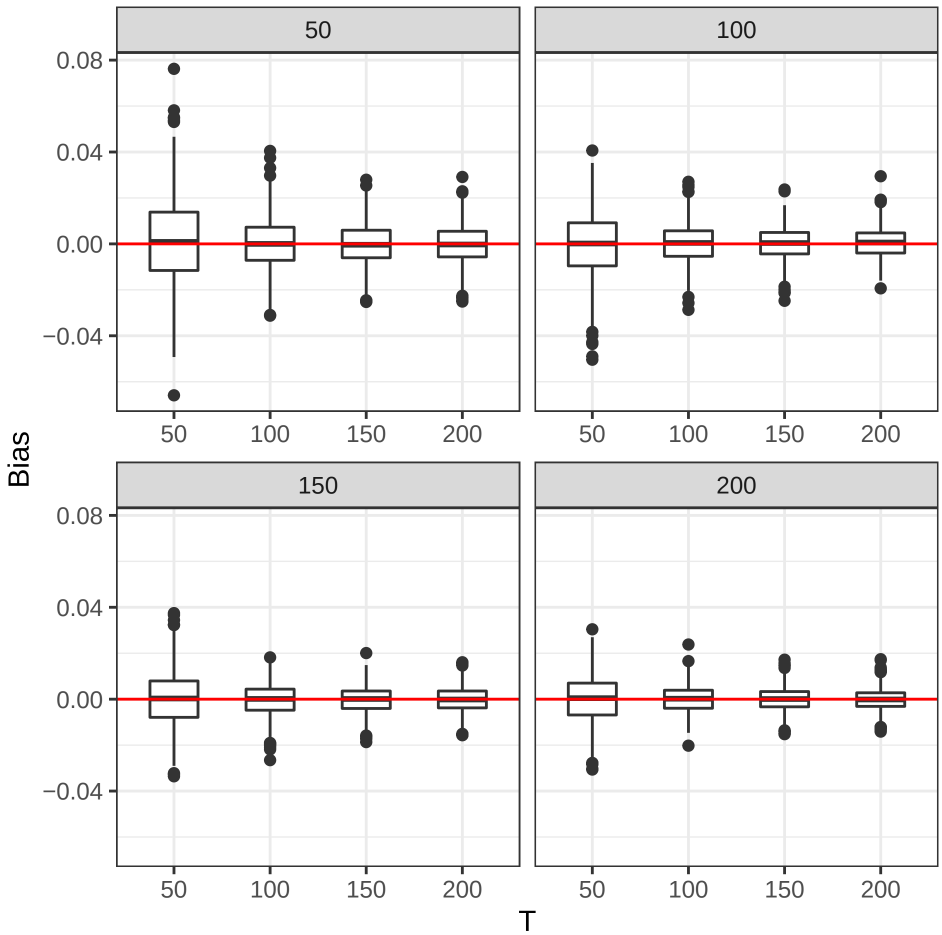
Notes: The figures display the distribution of the difference in absolute biases of the parameter $\beta^{(1)}$ for the estimation methods LCCE relative to CCE. Figures lower than zero mean that LCCE has lower absolute bias than CCE. The panel header show the number of cross-sections

Figure S4: Bias Comparison in $\beta^{(2)}$; $q = 4$



Notes: The figures display the distribution of the difference in absolute biases of the parameter $\beta^{(2)}$ for the estimation methods LCCE relative to CCE. Figures lower than zero mean that LCCE has lower absolute bias than CCE.The panel header show the number of cross-sections

Figure S5: Bias Comparison in $\beta^{(3)}$; $q = 4$



Notes: The figures display the distribution of the difference in absolute biases of the parameter $\beta^{(3)}$ for the estimation methods LCCE relative to CCE. Figures lower than zero mean that LCCE has lower absolute bias than CCE.The panel header show the number of cross-sections

Table S2: Simulation Results - Forecast Accuracy of PMIDAS Relative to AR(1) Benchmark - $q = 4$

	20 %				**30 %**				**50 %**			
N/T	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
50	0.8882	0.7482	0.7106	0.6898	0.9331	0.7612	0.7124	0.6957	1.1318	0.7791	0.7266	0.7048
100	0.8905	0.7459	0.7133	0.6899	0.9363	0.7594	0.7146	0.6963	1.1351	0.7760	0.7276	0.7049
150	0.8837	0.7480	0.7110	0.6885	0.9311	0.7610	0.7128	0.6939	1.1311	0.7772	0.7276	0.7036
200	0.8853	0.7491	0.7118	0.6911	0.9316	0.7608	0.7144	0.6952	1.1291	0.7772	0.7294	0.7043

Notes: This table represents the relative RMSFE of the simulated mixed-frequency data, resembling a monthly to weekly (or annual to quarterly) frequency mix. Figures less than one indicate superior performance of the PMIDAS model, as compared to a time series AR(1) model. Results are shown for sample split ratios for P equal to 20, 30 and 50 percent.

S3 Empirical Application I: Robustness to Additional Predictor Results

Figure S6: Additional Predictors; Target : Y-o-Y

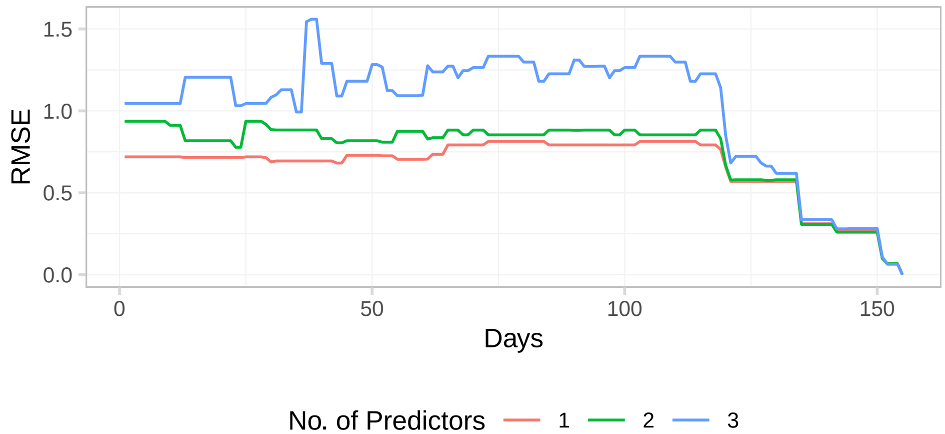
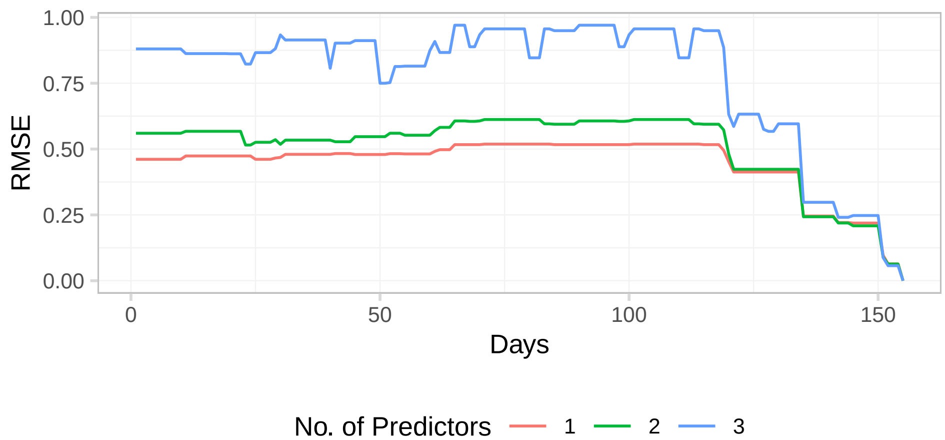


Figure S7: Additional Predictors; Target : Q-o-Q



S4 Empirical Application I: Robustness to Sample Split Results

Figure S8: 20 % split; Target : Y-o-Y

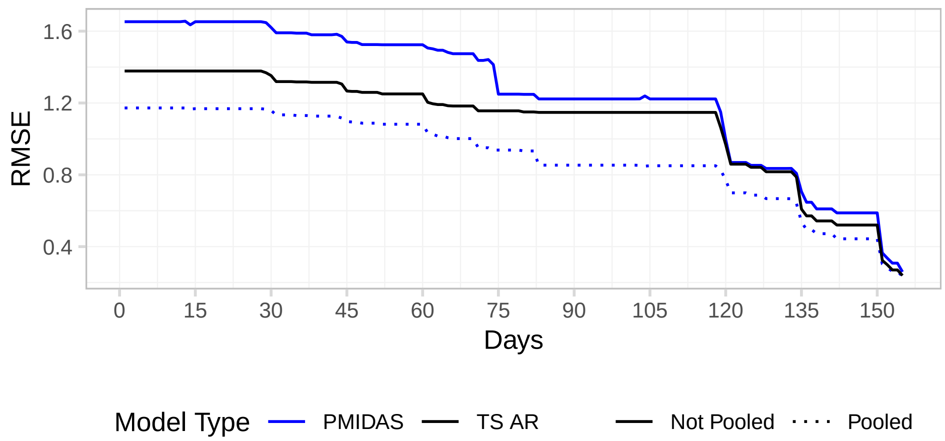


Figure S9: 40 % split; Target : Y-o-Y

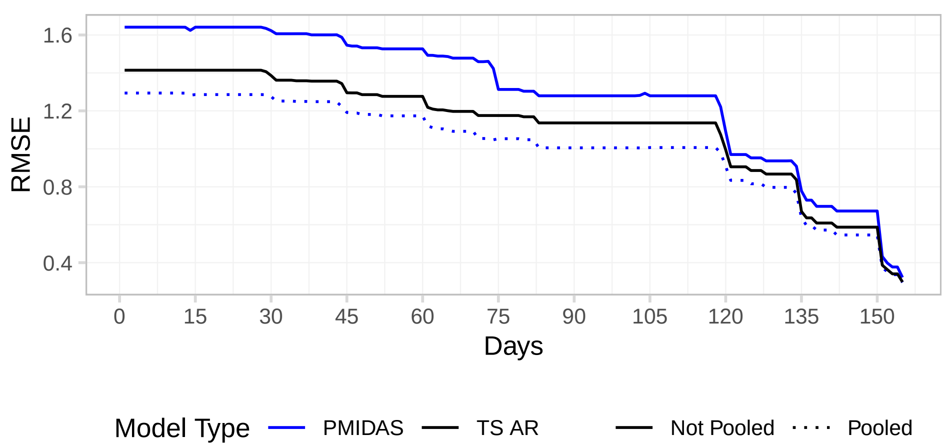


Figure S10: 20 % split; Target : Q-o-Q

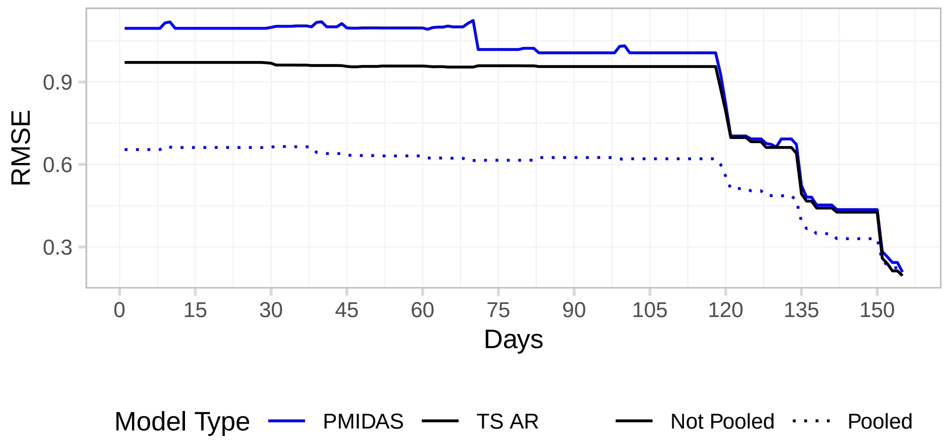


Figure S11: 40 % split; Target : Q-o-Q