Faculty of Natural, Mathematical, and Engineering Sciences
Department of Mathematics

PhD Projects in Mathematics

Applied Mathematics Research: Disordered Systems/Financial Mathematics/Probability/Theoretical Physics MPhil/PhD Projects

Project: **Understanding biodiversity in large ecosystems**

Supervisor: Izaak Neri

Group: Disordered Systems

Summary: What determines biodiversity is a longstanding question in theoretical ecology. Ecosystems can be described in terms of large networks of species, represented by nodes, and trophic interactions (who eats whom), represented by links; the picture shows the foodweb of the North American Pacific estuary in Baja California. The aim of this project is to develop methods to study dynamical systems defined on complex networks, and subsequently to use these methods to understand how foodwebs emerge in large ecosystems.

Project: **Collective pension investment**

Supervisor: John Armstrong

Group: Financial Mathematics

Summary: How can a group of people who face different risks collaborate to get better outcomes in retirement and how can they share the results of the investment in a way that is fair?

This project will develop the mathematical tools needed to define the optimal fair outcome in retirement, together with the computational tools needed to compute this in practice. The project as a whole requires innovative techniques from mathematics, economics and computing. The computational aspect will apply tools from machine learning to solve high dimensional partial differential equations.

It is not expected that an applicant will have all the skills required to solve both the mathematical and computational problems, the project will be a team effort.

Project: **Cylindridcal Levy processes**

Supervisor: Markus Riedle

Group: Probability/ Financial Mathematics

Summary: Cylindridcal Lévy processes are a new approach to model random perturbations of complex dynamical systems such as partial differential equations. It has already been observed that these cause completely different behaviour of the dynamical system than if perturbed by a classical stochastic process. In this project we investigate one of the many aspects which are unknown for cylindrical Levy processes, such as stability, ergodicity etc.

Project: **Microstructure of the Kerr black hole**

Supervisor: Dionysios Anninos

Group: Theoretical Physics

Summary: We will apply tools from modern theoretical physics to assess mathematical properties of highly spinning black holes. Such highly spinning black holes have been observed in our own Universe and display remarkable geometric features.

Project: **Exploring Secret Corners in the Space of Conformal Field Theories**

Supervisor: Andreas Stergiou

Group: Theoretical Physics

Summary: The space of conformal field theories contains many secret corners that remain unexplored. Your aim will be to use the most modern perturbative and non-perturbative methods to look into these corners, discover new theories and construct a more complete map of the space of conformal field theories.

Project: Lorentzian and Euclidean Conformal Field Theory

Supervisor: Petr Kravchuk

Group: Theoretical Physics

Summary: Conformal field theories have many facets, and appear in physics in both Lorentzian (or real-time), and Euclidean (or statistical) versions. From the theory perspective the Lorentzian point of view brings powerful analytical insights, while the Euclidean one allows for a precise numerical analysis. You will study the interplay between these two worlds to obtain new results relevant both in condensed-matter and high-energy physics.

Pure Mathematics Research MPhil/PhD Projects

Project: Foliations of 3-dimensional manifolds

Supervisor: Mehdi Yazdi

Group: Geometry

Summary: The student is expected to work in low-dimensional topology; in one of the areas of foliations, 3-dimensional manifolds, knot theory, mapping class groups of surfaces, or related subjects. Below some of these terms are briefly explained:

A foliation of a 3-dimensional manifold is a decomposition of it into disjoint surfaces that locally fit together like a stack of papers. Examples of singular foliations in one dimension lower are marbleised papers and tree rings. The student can for example study an important class of foliations called taut foliations, which has applications to knot theory.

A knot is an embedding of the circle in a 3-dimensional manifold, considered up to isotopy. In other words, we are allowed to deform the knot without the knot crossing itself. Knots have been extensively studied both for their own beauty, and as an important class of 3-dimensional manifolds (the complement of the knot).

The mapping class group of a surface is the group of homeomorphisms of the surface considered up to isotopy. This is an infinite group and has applications in various parts of geometry and topology including hyperbolic geometry of surfaces and the topology of 3-dimensional manifolds.

Project: **p-adic integration, fundamental groups and rational points**

Supervisor: Netan Dogra

Group: Number Theory

Summary: My research concerns trying to determine the rational number solutions to polynomial equations using a range of techniques from arithmetic algebraic geometry. In particular, using the etale topology, one can locate the rational number solutions inside the zeroes of certain p-adic transcendental functions constructed using the motivic fundamental group. This project involves studying these transcendental functions either from a theoretical perspective, e.g. trying to prove new theoretical results about solutions to certain families of equations, or from a computational perspective, developing algorithms to find all solutions to a Diophantine equation.

This project includes additional funding for travel in the first year

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Project: **Analytic aspects of L-functions and automorphic forms**

Supervisor: Stephen Lester

Group: Number Theory

Summary: L-functions and automorphic forms appear prominently in modern number theory and also arise in other areas of mathematics. The analytic properties of these functions are closely related to arithmetic problems involving the distribution of the prime numbers, lattice points, and structure of elliptic curves.

This project aims to further understand the analytic behaviour of L-functions and automorphic forms, with applications to problems arising in number theory or mathematical physics. Specific problems will follow the interests of the student and possible topics include: the distribution of zeros of Hecke cusp forms, moment estimates for families of L-functions, distribution of L^2-mass of Laplace eigenfunctions, and the behaviour of Fourier coefficients of automorphic forms.

Project: **Obstructions to local-global principles in families**

Supervisor: Rachel Newton

Group: Number Theory

Summary: The Hasse principle holds in a family of algebraic varieties if the existence of points over the reals and over the p-adic numbers for all primes p guarantees the existence of rational points on those varieties. This project will study obstructions to the Hasse principle in families of algebraic varieties.

Project: **Random waves on the triangle**

Supervisor: Igor Wigman

Group: Number Theory

Summary: This project concern the mysterious patterns appearing on musical instruments (and more generally, membranes), called "Chladni patterns". In this project the student will develop new methods in number theory in order to address this problem in mathematical physics, for the particular case of triangular dynamic system.

Statistics Research MPhil/PhD Projects

Project: Stein thinning in practice, with application to cardiac modelling

Supervisor: Marina Riabiz

Group: Statistics

Summary: Markov Chain Monte Carlo (MCMC) methods form the main approach to sample from probability distributions with intractable normalizing constants, such as the parameter posterior in Bayesian computation. Whilst MCMC samples converge in the limit to the target distribution, in many real-world applications it is possible to draw only a finite number of samples, due to restrictions in computing budget. In this framework, Stein thinning is a recent method developed to optimally select a set of samples from an MCMC output of fixed length, by minimizing a measure of discrepancy between the empirical approximation produced and the target. This PhD project aims at exploring different research questions that are still open, in order to produce a robust version of the algorithm, including the selection of a weighted kernel function that jointly guarantees weak convergence and convergence of certain moments of the distribution, based on related work on control variates and optimization of the power of goodness of fit test. The project will be applied to challenging Bayesian inverse problems arising in cardiac electrophysiology.

Project: Statistical analysis of supersaturated split-plot experiments

Supervisor: Dr Kalliopi Mylona

Group: Statistics

Summary: The objective of this project is to develop a new general methodology for analysing informative experiments with both restricted randomisation and a large number of factors. Split-plot designs are very effective in reducing the cost of an experiment in the presence of hard-to-change factors and/or of two-stage processes. In addition, supersaturated designs (SSDs) is a large class of factorial designs which can be used for screening out the important factors from a large set of potentially active variables. The huge advantage of these designs is that they reduce the experimental cost drastically but their critical disadvantage is the confounding involved in the statistical analysis. The combination of split-plot designs with supersaturated designs is a relatively unexplored research area, and the statistical analysis of data from this type of experiments is still not efficient.

In this project, the use of Bayesian analysis methods, will be studied. Gilmour and Goos (2009) recommended a Bayesian analysis, using an informative prior distribution for the whole plot variance component and implement this by using Markov chain Monte Carlo sampling. The designs considered in this project have an additional complication compared to the designs used in Gilmour and Goos (2009) and this is supersaturated property. We are planning to study and adapt several statistical analysis methods that had been used in the literature of supersaturated designs, such as Dantzig selector (Candes and Tao, 2007), the LARS (Efron, Hastie, Johnstone and Tibshirany,2004) methods and the Bayesian methods: SSVS (Chipman, Hamada and Wu, 1997) and Bayesian LASSO (Park and Casella, 2008).

Project: Supersaturated multi-stratum experiments

Supervisor: Dr Kalliopi Mylona

Group: Statistics

Summary: The objective of this project is to develop a new general methodology for setting up and analysing informative experiments with both restricted randomisation and a large number of factors. As designs with restricted randomisation are often much more cost-efficient than completely randomised designs, they are extremely popular in industry for quality improvement experiments and for experimenting with new products or processes. Multi-stratum designs with more than two strata are very effective in reducing the cost of an experiment in the presence of different levels of hard-to-change factors and/or of multi-stage processes. In addition, supersaturated designs (SSDs) is a large class of factorial designs which can be used for screening out the important factors from a large set of potentially active variables. The huge advantage of these designs is that they reduce the experimental cost drastically but their critical disadvantage is the confounding involved in the statistical analysis. The goal is to combine the multi-stratum designs (more than two strata) with the supersaturated designs. This will give the flexibility to the experimenters to use the cost-efficient SSDs under restricted randomised situations that appear very often in industrial experimentation. The combination of these two well-known classes of designs is a relatively unexplored research area (see, Koh et al. [2013], Lee et al. [2009], Chatterjee et al. [2018]). General construction and analysis methods for supersaturated multi-stratum designs with more than two strata will allow experiments that can extract the maximum information from the data with the minimum time and cost constraints.