Bachelier Seminar London-Paris

11 and 12 March 2021
Online workshop
https://www.kcl.ac.uk/events/london-paris-bachelier-workshop-in-financial-mathematics

Program and abstracts

Schedule given in Paris time (London time + 1h).

Thursday March, 11th

14h00 – 14h35 : **Luitgard Veraart**, The London School of Economics, Department of Mathematics
When does portfolio compression reduce systemic risk?
14h35 – 15h10 : **Noufel Frikha**, Université de Paris, LPSM
Some new integration by parts formula for finance and their Monte Carlo simulation
15h10 – 15h30 : Teasers for the Poster session
15h30 – 16h15 : Poster sessions

16h15 – 16h30 : Coffee break

16h30 – 17h05 : **Iuliia Manziuk**, Ecole polytechnique, CMAP
Adaptive trading strategies across liquidity pools
17h05 – 17h40 : **Eyal Neuman**, Imperial College London, Department of Mathematics - *The Multiplicative Chaos of H=0 Fractional Brownian Fields as the Interface between Volatility Models*
17h45 – 19h00 : Discussions

Friday March, 12th

10h00 – 10h35 : **Cyril Benezet**, ENSIIE, Université Paris Saclay, LaMME
Simulation of factor copulas with given marginals and estimation of extreme risk measures.
10h35 – 11h10 : **Tomasz Kosmala**, Queen Mary University, Department of Mathematics
Markov risk mappings and risk-sensitive optimal stopping.

11h10 – 11h30 : Coffee break

11h30 – 12h05 : **Ryan Donnelly**, King's College London, Department of Mathematics
Optimal Trading with Differing Trade Signals
12h05 – 12h40 : **Sarah Kaakaï**, Le Mans Université, LMM
Optimal and sustainable Pay-As-You-Go pension policy with minimum pension guarantee

Abstracts

Cyril BENEZET (ENSIIE, Université Paris Saclay, LaMME)

Simulation of factor copulas with given marginals and estimation of extreme risk measures.

Abstract: In this work, we are interested in the computation of statistics of the form $E[g(X)]$ or $E[g(X)|A]$ for a rare event A, where X is a multidimensional random vector which we cannot directly simulate. In a factor copula with given marginals context, we introduce an algorithm by Markov Chain transform to approximately sample from X and compute the desired quantity. We give convergence results with convergence rates. Last, we provide an application to extreme risk management, where X is a vector of assets returns with heavy-tailed marginals. This is a joint work with Emmanuel Gobet and Rodrigo Targino.

Ryan DONNELLY (King's College London, Department of Mathematics)

Optimal Trading with Differing Trade Signals.

Abstract: We consider the problem of maximizing portfolio value when an agent has a subjective view on asset value which differs from the traded market price. The agent's trades will have a price impact which affect the price at which the asset is traded. In addition to the agent's trades affecting the market price, the agent may change his view on the asset's value if its difference from the market price persists. We also consider a situation of several agents interacting and trading simultaneously when they have a subjective view on the asset value. Two cases of the subjective views of agents are considered, one in which they all share the same information, and one in which they all have an individual signal correlated with price innovations. To study the large agent problem we take a mean-field game approach which remains tractable. After classifying the mean-field equilibrium we compute the cross-sectional distribution of agents' inventories and the dependence of price distribution on the amount of shared information among the agents.

Noufel FRIKHA (Université de Paris, LPSM)

Some new integration by parts formula for finance and their Monte Carlo simulation.

Abstract: In this talk, I will present some new integration by parts (IBP) formulae for the marginal law at a given time maturity of killed diffusions as well as a class of stochastic volatility models with unbounded drift. Relying on a perturbation argument for Markov processes, our formulae are based on a simple Markov chain evolving on a random time grid for which we develop a tailor-made Malliavin calculus. Though such formulae could be further analyzed to study fine properties of the associated densities, our main motivation lies in their numerical approximation. Indeed, we show that an unbiased Monte Carlo path simulation method directly stems from our formulae so that it can be used in order to numerically compute with optimal complexity option prices as well as their sensitivities with respect to the initial values, the so-called Greeks, namely the Delta and Vega, for a large class of non-smooth European payoff. Numerical results are proposed to illustrate the efficiency of the method.

This talk is based on two joint works: with Arturo Kohatsu-Higa (Ritsumeikan university) and Libo Li (New South Wales university) on the one hand, with Junchao Chen (université de Paris) and Houzhi Li (université de Paris) on the other hand.

Sarah KAAKAI (Le Mans Université, LMM)

Optimal and sustainable Pay-As-You-Go pension policy with minimum pension guarantee.

Abstract: Intergenerational solidarity is one of the main pillars of pay-as-you-go (PAYG) pension plans. In (unfunded) PAYG systems, contributions paid by working participants are redistributed

to current retirees, inducing risk sharing between generations. However, PAYG systems in ageing countries face serious challenges. In this context, the sustainability of Defined Benefits (DB) PAYG systems has become a key challenge for policymakers, while Defined Contribution (DC) plans have gained momentum in the past years. However, DC pension systems should provide adequate benefits for retirees.
In this paper, we study the optimal pension policy of a PAYG system where defined contributions are combined with a minimum pension guarantee, and the social planner can invest part of the contributions (or borrow) in a buffer fund, operating as a risk-sharing mechanism among generations. We propose and study the existence of an adaptative optimal policy design under sustainability and adequacy constraints.We work within the framework of consistent progressive utilities, which allows us to convey the complexity of the problem, by taking into account key phenomena such as the demographic risk and its evolution over time, the time and age dependence of agents preferences, as well as financial risks. Joint work with Caroline Hillairet and Mohamed Mrad.

Tomasz KOSMALA (Queen Mary University, Department of Mathematics)

Markov risk mappings and risk-sensitive optimal stopping.

Abstract: We introduce a Markov property for dynamic risk mappings (non-linear expectations) in discrete time. The Markov property has several equivalent formulations, similar to the case of the linear expectation and is satisfied by the standard measures of risk used in practice such as Value at Risk and Average Value at Risk. We analyze the dual representation for convex Markovian risk mappings and give an application to risk-sensitive optimal stopping problems with intermediate costs under model ambiguity.

Iuliia MANZIUK (Ecole polytechnique, CMAP)

Adaptive trading strategies across liquidity pools.

Abstract: In this work, we provide a flexible framework for optimal trading in an asset listed on different venues. We take into account the dependencies between the imbalance and spread of the venues, and allow for partial execution of limit orders at different limits as well as market orders. We present a Bayesian update of the model parameters to take into account possibly changing market conditions and propose extensions to include short/long trading signals, market impact or hidden liquidity. To solve the stochastic control problem of the trader we apply the finite difference method and also develop a deep reinforcement learning algorithm allowing to consider more complex settings.

Eyal NEUMAN (Imperial College London, Department of Mathematics)

The Multiplicative Chaos of H=0 Fractional Brownian Fields as the Interface between Volatility Models.

Abstract:We consider a family of fractional Brownian fields $\{B^H\}_{H\in(0,1)}$ on $\mathbb{R}^d$, where H denotes their Hurst parameter. We first define a rich class of normalizing kernels ψ such that the covariance of

$$X^H(x) = \Gamma(H)^{\frac{1}{2}} \left(B^H(x) - \int_{\mathbb{R}^d} B^H(u)\psi(u,x)du \right),$$

converges to the covariance of a log-correlated Gaussian field when $H \downarrow 0$.

We then use Berestycki's "good points" approach in order to derive the limiting measure of the so-called *multiplicative chaos of the fractional Brownian field*

$$M^H_\gamma(dx) = e^{\gamma X^H(x) - \frac{\gamma^2}{2}E[X^H(x)^2]}dx,$$

as $H \downarrow 0$ for all $\gamma \in (0, \gamma^*(d)]$, where $\gamma^*(d) > \sqrt{\frac{7}{4}d}$. As a corollary we establish the L^2 convergence of M_γ^H over the sets of "good points", where the field X^H has a typical behaviour. As a by-product of the convergence result, we prove that for log-normal rough volatility models with small Hurst parameter, the volatility process is supported on the sets of "good points" with probability close to 1. Moreover, on these sets the volatility converges in L^2 to the volatility of multifractal random walks. This is a joint work with Paul Hager.

Luitgard VERAART (London School of Economics, Department of Mathematics)

When does portfolio compression reduce systemic risk?

Abstract: We analyse the consequences of portfolio compression on systemic risk. Portfolio compression is a post-trading netting mechanism that reduces gross positions while keeping net positions unchanged and it is part of the financial legislation in the US (Dodd-Frank Act) and in Europe (European Market Infrastructure Regulation). We derive necessary structural conditions for portfolio compression to be harmful and discuss policy implications. In particular, we show that the potential danger of portfolio compression comes from defaults of firms that conduct portfolio compression. If no defaults occur among those firms that engage in compression, then portfolio compression always reduces systemic risk.

Poster sessions

Duc-Thinh VU (Paris Dauphine University, Ceremade)

Consistent Risk Measure on L^0: NA Condition, Pricing and Dual Representation

Abstract: The NA condition is one of the pillars supporting the classical theory of financial mathematics. We revisit this condition for financial market models where a dynamic risk-measure defined on L^0 is fixed to characterize the family of acceptable wealths that play the role of non-negative financial positions. We provide in this setting a new version of the fundamental theorem of asset pricing and we deduce a dual characterization of the super-hedging prices of a European option. Moreover, we provide an example where it is possible to obtain a dual representation of the risk-measure on L^0. This is joint work with Emmanuel Lepinette.

Guillaume SZULDA (Université de Paris, LPSM)

Multiple yield curve modelling with CBI processes

Abstract: We develop a modelling framework for multiple yield curves driven by continuous-state branching processes with immigration (CBI processes). Exploiting the self-exciting behavior of CBI jump processes, this approach can reproduce the relevant empirical features of spreads between different interbank rates. In particular, we introduce multi-curve models driven by a flow of tempered alpha-stable CBI processes. Such models are especially parsimonious and tractable, and can generate contagion effects among different spreads. We provide a complete analytical framework, including a detailed study of discounted exponential moments of CBI processes. The proposed approach allows for explicit valuation formulae for all linear interest rate derivatives and semi-closed formulae for non-linear derivatives via Fourier techniques and quantization. We show that a simple specification of the model can be successfully calibrated to market data. This is joint work with Claudio Fontana (University of Padova) and Alessandro Gnoatto (University of Verona).

Marcos LEUTSCHER (Ensae Paris, CREST)

Control and optimal stopping Mean Field Games: a linear programming approach

Abstract: We develop the linear programming approach to mean-field games in a general setting. This relaxed control approach allows to prove existence results under weak assumptions, and lends itself well to numerical implementation. We consider mean-field game problems where the representative agent chooses both the optimal control and the optimal time to exit the game, where the instantaneous reward function and the coefficients of the state process may depend on the distribution of the other agents. Furthermore, we establish the equivalence between mean-field games equilibria obtained by the linear programming approach and the ones obtained via the controlled/stopped martingale approach, another relaxation method used in a few previous papers in the case when there is only control. This is joint work with Roxana Dumitrescu (Kings College London) and Peter Tankov (Institut Polytechnique de Paris).

Florian BOURGEY (Ecole polytechnique, CMAP)

Weak approximations and VIX option prices expansions in rough forward variances models

Abstract: We provide approximation formulas for VIX derivatives in forward variance models, with particular emphasis on the family of so-called Bergomi models: the rough Bergomi model of Bayer, Friz, and Gatheral (2016), and an enhanced version that can generate realistic positive skew for VIX smiles - introduced simultaneously by De Marco (2018) and Guyon (2018) mimicking the ideas of Bergomi (2008), that we refer to as 'mixed rough Bergomi model'. Following the methodology of Gobet and Miri (2014), we derive weak approximations for the VIX random

variable, expressed as a combination of lognormal proxies, leading to option price approximations under the form of explicit combinations of Black–Scholes prices and greeks. We stress that our approach does not rely on small-time asymptotics and can, therefore, be applied to any option maturity. Our results are illustrated by several numerical experiments and calibrations to VIX market data. This is joint work with Stefano De Marco (Ecole Polytechnique) and Emmanuel Gobet (Ecole Polytechnique).

Tim KING (King's College London, Department of Mathematics)

Curved Schemes for SDEs on Manifolds

Abstract: Given a stochastic differential equation (SDE) in $\mathbb{R}^n$ whose solution is constrained to lie in some manifold $M \subset \mathbb{R}^n$, we propose a class of numerical schemes for the SDE whose iterates remain close to M to high order. Our schemes are geometrically invariant, and can be chosen to give perfect solutions for any SDE which is diffeomorphic to n-dimensional Brownian motion. Unlike projection-based methods, our schemes may be implemented without explicit knowledge of M. Our approach does not require simulating any iterated Itô interals beyond those needed to implement the Euler-Maryuama scheme. We state a result on the convergence of the scheme, and give some experiments showing that the schemes perform well in practice; these include the stochastic Duffing oscillator, the stochastic Kepler problem, and also an application to Markov chain Monte Carlo. This is joint work with John Armstrong (King's College London).

Andrei IONESCU (King's College London, Department of Mathematics)

Pathwise robust gamma hedging using reduced rough paths

Abstract: We derive a fundamental theorem of derivative trading in the framework of rough paths. We show that, given market option prices are well approximated by the trader's model, they can robustly hedge European options path wise. We show that the classical continuous delta hedging strategy is a perfect and robust hedge but that its financial interpretation breaks down path wise. Instead, we show that continuous gamma hedging is the perfect, robust hedge in a path wise sense. This is joint work with John Armstrong (King's College London).

Zexin WANG (Imperial College London, Department of Mathematics)

Optimal Liquidation with Hidden Orders under Self-Exciting Market Order Dynamics

Abstract: We develop an optimal execution strategy for an agent seeking to liquidate her position within a fixed time horizon using both hidden and limit orders in a lit pool. We formulate the execution task in a continuous-time framework with inhomogeneous point process for market order arrivals. Under a reduced simple case where arrivals follow homogeneous Poisson process, we derive a quasi-closed-form solution. We show in this case there is a cutoff time, before which the agent should enter a pure-hidden-order phase, and switch into a mixed-orders phase until termination. On the other hand, when arrivals follow Hawkes process, numerical solution is derived with feedback controls. We show in the case with self-exciting orders, that there is a cutoff surface determined by market condition and agents time pressure, based on which the agent continuously monitors the real-time market activity and changes her strategies. In particular, the optimal hidden order size increases with the time pressure of agent under the pure-hidden-order phase, and decreases with time pressure under the mixed-orders phase, whereas the sum of limit and hidden order always increases with time pressure. We show how the execution strategies perform when comparing with pure limit order strategy in simulations. It suggests that taking into account hidden orders significantly reduces liquidation cost. Real data analysis shows that the mixed order strategy achieves a 9.3% reduction in execution cost when adopting the self-exciting assumption for the

market order arrivals. We attribute the improvement over alternatives to the optimal mix of hidden and limit orders, which earns the spread, increases the total liquidity provision in the market, keeping the agents liquidation schedule on target. This is joint work with Ying Chen (National University of Singapore), Ge Zhang (NUS Graduate School for Integrative Sciences and Engineering and National University of Singapore) and Chao Zhou (City University of Hong Kong).

Maxime GRANGEREAU (CMAP Ecole polytechnique and EDF R&D)

Federated stochastic control of numerous heterogeneous energy storage systems

Abstract: We propose a stochastic control problem to manage cooperatively heterogeneous Thermostatically Controlled Loads (TCLs) to promote power balance in electricity networks. We develop a method to solve this problem with a decentralized architecture, in order to respect privacy of individual users and to reduce both the telecommunications and the computational burden compared to the setting of an omniscient central planner. This paradigm is called federated learning in the machine learning community, hence the name federated stochastic control problem. The optimality conditions are expressed in the form of a high-dimensional Forward-Backward Stochastic Differential Equation (FBSDE), which is decomposed into smaller FBSDEs, which fully characterize the equilibrium of a stochastic differential game. In this game, a coordinator (the leader) aims at controlling the aggregate behavior of the population, by sending appropriate signals, and agents (the followers) respond to this signal by optimizing their storage system locally. A mean-field-type approximation is proposed to circumvent telecommunication constraints and privacy issues. Convergence results and error bounds depending on the size of the population are obtained for this approximation. Numerical experiments show the interest of the control scheme and to exhibit the convergence of the approximation. An implementation which answers practical industrial challenges to deploy such a scheme is developed. This is joint work with Emmanuel Gobet (CMAP Ecole polytechnique).

Laura TINSI (EDF Lab and CREST ENSAE Paris)

Sequential decision making with probabilistic forecasts

Abstract: We consider a sequential decision making process (trading, investment, production scheduling etc.) whose outcome depends on the realization of a random factor, such as a meteorological variable. We assume that the decision maker disposes of a probabilistic forecast (predictive distribution) of the random factor, which is regularly updated. We propose several stochastic models for the evolution of the probabilistic forecast, and show how these models may be estimated from the historical forecast data. We then show how this stochastic model can be used to determine optimal decision making strategies depending on the forecast updates. Applications to wind energy trading are given. This is joint work with Peter Tankov (CREST ENSAE IPP).

Zacharia ISSA (King's College London, Department of Mathematics)

An Optimal Transport Approach to Market Regime Clustering

Abstract: The problem of rapid and automated detection of distinct market regimes is a topic of great interest to financial mathematicians and practitioners alike. In this paper, we outline an unsupervised learning algorithm clusters a given time-series - corresponding to an asset or index - into a suitable number of temporal segments (market regimes).

In particular, we develop an algorithm for this purpose, that permits to automate the process of identification of market regimes on the space of multivariate probability measures with finite pth moment. This method - the principle of which is inspired by the well-known k-means algorithm - clusters said segments on the space of probability measures with finite p^{th} moment. On this

space, our choice of metric is the p-Wasserstein distance. We compare our Wasserstein-kmeans approach with a more traditional implementation of the k-means algorithm by generating clusters in Euclidean space via the first N raw moments of each log-return segment instead (moment-k-means).
We compare the two approaches initially on real data and validate the performance of either algorithm by studying the maximum mean discrepancy between, and within, clusters. In both cases it is shown that the Wasserstein-k-means algorithm vastly outperforms the moment-based approach. We also test both algorithms on synthetic data, and in particular study how close the centroids obtained from either algorithm are to the true distributions associated to each regime. Here, we see that the Wasserstein-k-means algorithm performs well, even in the case where true distributions are not Gaussian. This is joint work with Blanka Horvath (Kings College London, Imperial College London, Alan Turing Institute) and Aitor Muguruza (Imperial College London, Kaiju Capital Management).

Zineb El FILALI ECH-CHAFIQ (Université Grenoble Alpes, CNRS and Quantitative Analyst at Natixis)

Automatic Control Variates for option pricing using Neural Networks

Abstract: Many pricing problems boil down to the computation of a high-dimensional integral, which is usually estimated using Monte Carlo. In fact, the accuracy of a Monte Carlo estimator with M simulations is given by $\sigma\sqrt{M}$. Meaning that its convergence is immune to the dimension of the problem. However, this convergence can be relatively slow depending on the variance σ of the function to be integrated. To resolve such a problem, one would perform some variance reduction techniques such as importance sampling, stratification, or control variates. In this paper, we will study two approaches for improving the convergence of Monte Carlo using Neural Networks. The first approach relies on the fact that many high-dimensional financial problems are of low effective dimensions. We expose a method to reduce the dimension of such problems in order to keep only the necessary variables. The integration can then be done using fast numerical integration techniques such as Gaussian quadrature. The second approach consists in building an automatic control variate using neural networks. We learn the function to be integrated (which incorporates the diffusion model plus the payoff function) in order to build a network that is highly correlated to it. As the network that we use can be integrated exactly, we can use it as a control variate. This is joint work with Jérôme Lelong (Université Grenoble Alpes, CNRS) and Adil Reghai (Head of Quantitative Research, Equity and Commodity Markets, Natixis).

Ofelia BONESINI (University of Padova, Dipartimento Di Matematica)

Functional Quantization of rough volatility and applications to the VIX

Abstract: We develop here a product functional quantization of rough volatility. Because the optimal quantizers can be computed offline, this new technique, built on the insightful works by Luschgy and Pagès [Luschgy, H. and Pagès, G. (2002): Functional quantization of Gaussian processes, Journal of Functional Analysis, 196, pp. 486-531.; Luschgy, H. and Pagès, G. (2007): High-resolution product quantization for Gaussian processes under sup-norm distortion, Bernoulli, 13(3), pp. 653-671; Pagès, G. (2007): Quadratic optimal functional quantization of stochastic processes and numerical applications, Monte Carlo and Quasi-Monte Carlo Methods 2006, Springer.], becomes a strong competitor in the new arena of numerical tools for rough volatility. We concentrate our numerical analysis to pricing VIX futures in the rough Bergomi model [Bayer, C.; Friz, P.K. and Gatheral, J. (2016): Pricing under rough volatility, Quantitative Finance, 16(6), pp. 887-904.] and compare our results to other benchmarks recently suggested. This is joint work with

Giorgia Callegaro (University of Padova) and Antoine Jacquier (Imperial College London).