PhD Projects in Mathematics 2024-25

Applied Maths Research

Project: Kasteleyn theory

Supervisor: Sam Johnston

Group: Probability

Summary: Kasteleyn theory is a tool used by probabilists to obtain explicit solutions for the partition functions and correlation functions for models of random surfaces and random tilings. In this project we use a new coordinate system to study generalisations of these models in higher dimensions, making connections with areas of probability, algebra, and statistical physics.

Project: Cylindridcal Levy processes

Supervisor: Markus Riedle

Group: Probability/ Financial Mathematics

Summary: Cylindridcal Lévy processes are a new approach to model random perturbations of complex dynamical systems such as partial differential equations. It has already been observed that these cause completely different behaviour of the dynamical system than if perturbed by a classical stochastic process. In this project we investigate one of the many aspects which are unknown for cylindrical Levy processes, such as stability, ergodicity etc.

Project: Entanglement and universality in inhomogeneous and out-of-equilibrium many body quantum systems

Supervisor: Paola Ruggiero & Benjamin Doyon

Group: Disordered Systems

Summary: The project lies at the intersection of several different fields: from statistical physics to quantum information, from high energy to condensed matter physics. The common denominators are the phenomenon of quantum entanglement and the out-of-equilibrium behaviour of many-body quantum systems, particularly in low dimensional quantum many body systems. While the emergence of universality is rather well established for systems which are homogeneous and are at equilibrium, the same is not true for inhomogeneous systems, both in and out of equilibrium. Those, on the other hand, are the rule rather than the exception in the context of quantum experiments. Different new theories and tools are currently under study to investigate what remains of the aforementioned universality in such more complicated situations, and the projects aims at going deeper in this direction, mainly using the “lens” of quantum entanglement.

Project: The mathematics of adaptive, stable and robust Artificial Intelligence

Supervisor: Ivan Tyukin

Group: Disordered Systems

Summary: Recent years have seen explosive growth in the applications of Artificial Intelligence (AI). Notwithstanding significant successes of the new technology, empirical evidence points to numerous examples of errors in the decisions of state-of-the-art AI systems. Current theoretical works confirm that modern design practices and classical mathematical frameworks supporting these could suffer from various fundamental shortcomings including the typicality of adversarial attacks, susceptibility to backdoors and stealth attacks, instabilities, and hallucinations. At the same time, there is a plethora of under-explored and seemingly controversial phenomena, such as learning from scarce yet high-dimensional information, which has been broadly observed experimentally in large-scale AI systems but whose complete mathematical understanding is yet to be developed. Other factors hindering the performance of current AI systems include the need to adapt to concept drifts and unforeseen changes in operational conditions. The project aims to explore and systematically assess major causes of AI errors and instabilities and develop appropriate mathematical foundations enabling the creation of stable and robust Artificial Intelligence capable of dealing with errors and concept drifts. The successful candidate will work alongside a team of early career researchers funded by the UKRI Turing AI Acceleration Fellowship (EP/V025295/2) and will benefit from the team’s established relationships with industrial partners, academic networks (Turing AI Fellows, Trustworthy Autonomous Systems Node in Verifiability EP/V026801/2), and other collaborators in academia, healthcare, defence & security.

Project title : Random walks in dynamic random environments

Supervisor: Guillaume Conchon-Kerjan

Group: Probability

Summary: Random walks in dynamic environments are a class of models, at the boundary of discrete probability and statistical physics, in which a particle is advected by a fluid whose evolution is random. With traps constantly forming and dissolving, classical methods from static environments (e.g. potential theory) become inefficient, and understanding the fluctuations of the random walker on this evolving landscape has attracted a lot of attention recently.
From the probability theory point of view, some of the main challenges to date are: adding a current flow to the environment, cooling down the environment, or making the environment spatially inhomogeneous.

This project aims to address one or several of them (to be discussed with the candidate). It will blend tools from several areas of probability such as percolation, mixing times and interacting particle systems.

Project: Exploring the non-equilibrium states of matter

Supervisor: François Huveneers

Group: Disordered Systems

Summary: Context: Equilibrium statistical mechanics offers a precise framework for understanding macroscopic physical systems in their long-term state, characterized by maximal entropy for given values of globally conserved quantities. This simple precept allows to determine precisely the equilibrium states. However, the most captivating and profound phenomena in nature often manifest in systems that have not yet attained equilibrium, or that are driven out of it through some external forcing. In these instances, we lack any fundamental guiding principle that would permit an accurate description of matter at the macroscopic level. Consequently, we find ourselves exploring specific examples in a quest for a comprehensive theory.

Project: The primary objective of this Ph.D. project is to delve into some of these examples using the tools of theoretical physics, mathematical physics and probability theory, with the emphasis on a precise and detailed understanding. Disordered interacting quantum systems offer a particularly rich phenomenology. In particular, the interplay between disorder and interactions may lead to the so called many-body localized phase where the approach to equilibrium is halted by emergent conserved quantities. The physical characteristics of this phase have been explored intensively since more than a decade and the field is probably reaching maturity from a phenomenological point of view. In contrast, the mathematical understanding remains very incomplete, reflecting into the fact that several fundamental aspects are still intensely debated. There is thus a clear need for the development of a better mathematical theory and the search for new techniques. A direct goal of this project will be to bring the theory on much firmer grounds. Moreover, from a broader perspective, the hope is that these contributions eventually lead to a more profound and cohesive understanding of non-equilibrium states of matter as a whole.

I seek for a very motivated student with a solid background in either applied mathematics or theoretical physics. Most importantly, the successful candidate is expected to have some taste for deriving rigorous mathematical results..

Pure Maths Research

Project: Analytic aspects of L-functions and automorphic forms

Supervisor: Stephen Lester

Group: Number Theory

Summary: L-functions and automorphic forms appear prominently in modern number theory and also arise in other areas of mathematics. The analytic properties of these functions are closely related to arithmetic problems involving the distribution of the prime numbers, lattice points, and structure of elliptic curves. For instance, Millennium prize problems such as the Generalised Riemann Hypothesis and Birch and Swinnerton-Dyer Conjecture are directly related to the theory of these functions. This project aims to further understand the analytic behaviour of L-functions and automorphic forms, with applications to problems arising in number theory or mathematical physics. Specific problems will follow the interests of the student and possible topics include: the distribution of zeros of Hecke cusp forms, moment estimates for families of L-functions, and the behaviour of Fourier coefficients of automorphic forms

Project: Birational Geometry and foliations

Supervisor: Calum Spicer

Group: Geometry (Pure mathematics)

Summary: The goal of this project is to explore cutting edge applications and interactions between birational geometry and holomorphic foliation theory. There is a specific focus on the use of the Minimal Model Program and related techniques as applied to questions in local and global geometry of algebraic varieties and foliations. The study of the Minimal Model Program is one of the main research directions in modern algebraic geometry, and in recent years has seen many significant developments. Among these is a new focus on relations between the Minimal Model Program and foliations, which has resulted in applications of ideas and techniques of the Minimal Model Program to a wide range of novel situations.

Statistics Research

Supervisor: Davide Pigoli

Group: Statistics, Stochastic Processes, Mathematical Modelling.

Summary: There is a growing interest in the statistical analysis of data that take value on smooth manifolds (3D rotations, shapes, directions, positive definite matrices, points on a circle or a sphere, constrained curves to mention just a few examples). The goal of the project is to develop a framework to analyse manifold-valued data that are observed over time and/or space, example including molecular dynamics simulations of protein folding in medicinal chemistry (temporally dependent), permeability tensors in mining engineering (spatially dependent) or coordinates (latitude and longitude) of earthquakes aftershocks in earth science (spatio-temporally dependent). Possible lines of research include (but are not limited to): - modelling of stochastic dependence in these non-linear spaces, where usual concepts of co-variability are not valid. - extension to dependent samples of existing techniques for dimensional reduction (Principal Geodesic Analysis, Principal Flows) and centerpoint estimation (Fréchet mean) for manifold-valued data. - multiresolution analysis and prediction for spatio-temporal manifold-valued processes.

Project: Changepoint detection in Non-Euclidean spaces

Supervisor: Davide Pigoli

Group: Statistics, Stochastic Processes, Applied Statistics.

Summary: This project is concerned with the development and analysis of cumulative sums statistics (Barnard, 1959) to detect changepoints in sequences where the observations belong to a non-Euclidean space (Chen, 2019). Examples include time series of directional data (e.g., wind directions), time series of correlation matrices (e.g., correlation matrices between electroencephalogram signals) or dynamical networks (e.g., social networks or trade networks). Cumulative sum statistics can be defined based on a metric in the non-Euclidean space and there are already some results for data on a circle (Lombard et al., 2020). The purpose of this project is to extend this approach to more general non-Euclidean spaces, and analyse the performance of changepoint detection procedures based on these statistics both theoretically and in practice.

Project: Stein thinning in practice, with application to cardiac modelling

Supervisor: Marina Riabiz

Group: Statistics

Summary: Markov Chain Monte Carlo (MCMC) methods form the main approach to sample from probability distributions with intractable normalizing constants, such as the parameter posterior in Bayesian computation. Whilst MCMC samples converge in the limit to the target distribution, in many realworld applications it is possible to draw only a finite number of samples, due to restrictions in computing budget. In this framework, Stein thinning is a recent method developed to optimally select a set of samples from an MCMC output of fixed length, by minimizing a measure of discrepancy between the empirical approximation produced and the target. This PhD project aims at exploring different research questions that are still open, in order to produce a robust version of the algorithm, including the selection of a weighted kernel function that jointly guarantees weak convergence and convergence of certain moments of the distribution, based on related work on control variates and optimization of the power of goodness of fit test. The project will be applied to challenging Bayesian inverse problems arising in cardiac electrophysiology

Project: Designing experiments on big networks

Supervisor: Dr Vasiliki Koutra

Group: Statistics

Summary: We live in a hyper-connected world, with networks of all kinds used to represent different systems: social networks, financial networks, biological networks, geographical proximity networks and others. Experiments are now routinely performed to understand the impact of interventions on these networks in many areas of science, technology and industry. For example, for a marketing experiment on a social network, we need to select which users should receive which advertisements in order to assess differences in click-through rates or revenue. Formal methods for the optimal design of experiments on large and complex networks are in their infancy.

A substantial complication in networked experiments arises from the need to estimate both the direct effect of the treatment applied to each node and the indirect, or viral, effect of treatments applied to connected nodes [1, 2, 3]. In our social media marketing example, the click-through rate from a given user may be affected by advertisements allocated to other connected users on the network. For large networks, e.g. derived from social media, existing optimal design approaches are too computationally expensive to be employed in practicable time. Hence, heuristic and sub-optimal treatment selections are often applied. It is this gap that this project will address.

References:

[1] Parker, B.M., Gilmour, S.G., Schormans, J. (2017) JRSSC, 66, 455-480.

[2] Koutra, V., Gilmour, S.G., Parker, B.M. (2021) JRSSC, 70, 596-618.

[3] Koutra, V., Gilmour, S.G., Parker, B.M., Mead, A. (2023) JABES, 28, 526–548.

Project: Bayesian learning for partially specified models

Supervisor: Yu Luo

Group: Statistics

Summary: Bayesian inference shifts from unknown deterministic quantities to studying distributions, which has been proven increasing powerful in a large spectrum of applications over the decades. The posterior inference is often thought as a procedure to update prior beliefs in light of the data. However, in the usual Bayesian setting where a full probabilistic model is required. However, recently some researchers in statistical learning have investigated extensions of the Bayes paradigm to the data-driven procedure, which one does not make any assumption on the probabilistic model, but rather relies on an agnostic measure between the model prediction and outcome, resulting a partially specified model. These extensions no longer abide by the canonical Bayesian rules and may be harder to interpret by practitioners. Therefore, the overall goal of this project is to help develop a mature general Bayesian decision-making machinery for models which are partially specified. This methodology advancement has the potential to apply in various fields, such as machine learning and causal inference.

Project: Multi-objective Optimal Design of Experiments for Dimensional Analysis

Supervisor: Professor Steven Gilmour

Group: Statistics

Scientists and engineers often use the technique of dimensional analysis before conducting experiments, in order to identify dimensionless metrics on which to use both response variables and experimental factors. The aim is to obtain dimensionless quantities, so that relationships discovered experimentally can be extrapolated to different dimensions of the natural variables. When designing the experiments, we want to ensure that (i) the analysis of the dimensionless variables will be efficient; (ii) that we have protection against modest departures from the assumption of dimensionlessness; and (iii) that we are able to detect large departures from the assumption of dimensionlessness.

Albrecht et al. (2013) were the first to consider design of experiments for dimensional analysis. They suggested using multi-objective optimal designs and this work was later extended to multivariate responses by Eck et al. (2020) and to discrete responses by Woods et al. (2017). Although these authors identified the issues of robustness to assumptions and tried to choose designs which address this, their designs did not directly ensure efficiency for the data analyses which would actually be done.

In this project, we will tackle the problem more directly and will also ensure other issues of model robustness are considered. We will tackle the following issues.

1. Ensure designs are robust to lack of fit of the assumed model with respect to the dimensionless variables. This should be a simple extension of the work of Egorova and Gilmour (2022).

2. Consider lack of fit of the assumption of constant variance (on the measured scale). The usual theory of randomisation-based analysis of the data suggests a particular variance structure, but the transformation of variables to achieve dimensionless quantities might bring this assumption into question.

3. The lack of fit with respect to the assumption of dimensionlessness should be best tackled by nesting the assumed model in the dimensionless variables in a wider class of models, while keeping as much as possible of the dimensionless structure. Designing such experiments will need further developments of the ideas of Egorova and Gilmour (2022).

We are currently exploring possible collaborations with experimental scientists and engineers at King's.

References:

Albrecht, MC, Nachtsheim, CJ, Albrecht, TA and Cook, RD (2013) Experimental design for engineering dimensional analysis. Technometrics, 55, 257-270.

Echt, DJ, Cook, RD, Nachtsheim, CJ and Albrecht, TA (2020) Multivariate design of experiments for engineering dimensional analysis. Technometrics, 62, 6-20.

Egorova, O and Gilmour, SG (2022) Optimal response surface designs in the presence of model contamination. arxiv:2208.05366.

Woods, DC, Overstall, AM, Adamou, M and Waite, TW (2017) Bayesian design of experiments for generalized linear models and dimensional analysis with industrial and scientific application. Quality Engineering, 29, 91-103