Neutrino Oscillations Activity Worksheet - Solutions

In this activity you are going to have a go at classifying simulated events for the Super-Kamiokande experiment. You will then determine whether there is evidence of oscillation in your data set.

There are 10 data sets with 50 events in each – that is 500 events in total but you don't have to analyse all of them. One approach is for the class to split into groups and each group work on a different data set. Don't worry if you don't get through the whole set – just record the range of events you looked at and the total number.

The events are randomly distributed so you will see variation between the data sets, but the more data you examine, the more accurate your result should be so we suggest combining the results across groups at the end.

The solutions are in the following pages. When you have finished looking at these, please remember to fill out the feedback form.

Step 1 Watch the videos and then try to answer the following questions.

What ratio of muon-like to electron-like neutrinos do we expect (in the absence of neutrino oscillations)?

Your answer: 2. (ie 2 muon neutrinos for every 1 electron neutrino – here we don't need to differentiate between neutrinos and anti-neutrinos)

Which neutrinos travel further, upwards or downwards going ones?

Your answer: Upwards (they travel all the way across the Earth, downwards just come down from above)

Why might you expect the ratio to be different for upwards and downwards events?

Your answer: Not in the absence of oscillations. If oscillations happen, it should affect the upwards neutrinos more as they travel further

Step 2

Now watch the video which talks about neutrino event displays.

Using the online event display[1], work through a range of events and try to classify them into one of the five categories:

- Incoming background (hits in the outer detector)
- Muon-like (sharp-ring) Upwards going
- Muon-like (sharp-ring) Downwards going
- Electron-like (fuzzy ring) Upwards going
- Electron-like (fuzzy ring) Downwards going

There are 10 data sets with 50 events in each – that is 500 events in total - but don't worry, you don't have to analyse all of them. Just record the range of events you looked at and the total number.

Mark the data set, event range and total number of events you classify at the top of the table on page 3 (applicable if you are sharing the task across groups in the class).

Mark each Event ID number in the relevant box in the provided Results Table on page 3.

Step 3

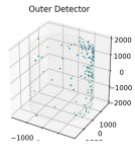
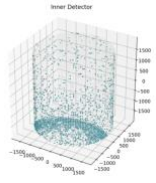
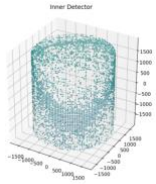
When you have finished count up the number of events in each classification and record it in the Results Table.

Step 4

Now calculate the ratio of muon to electron type for Downwards events and for Upwards events and record these at the bottom of your Results Table.

[1] URL: https://invisible-explorers.sites.er.kcl.ac.uk/

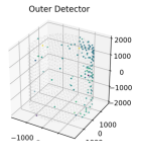
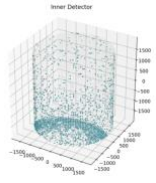
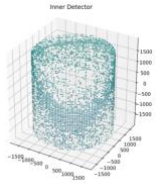
Results table – Data Set 1

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 0, 8, 17, 24, 34, 37, 41, 42, 44, 49
(hits in outer detector)
 
</td></tr>
<tr><td colspan="2"></td><td>Total background events: 10</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
  </td><td>Upwards: 4, 11, 12, 14, 22, 30, 31, 45, 46</td><td>Downwards: 1, 2, 13, 15, 20, 21, 25, 26, 27, 32, 36, 39, 47, 48</td></tr>
<tr><td>Total Upward muon events: 9</td><td>Total Downward muon events: 14</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
  </td><td>Upwards: 3, 5, 6, 7, 9, 10, 19, 28, 33, 35, 40, 43</td><td>Downwards: 16, 18, 23, 29, 38</td></tr>
<tr><td>Total Upward electron events: 12</td><td>Total Downward electron events: 5</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{14}{5} = 2.8$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{9}{12} = 0.75$$

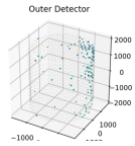
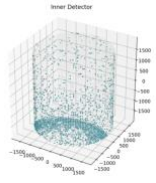
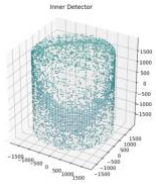
Results table – Data Set 2

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 13, 25, 26, 41
(hits in outer detector)
 
</td></tr>
<tr><td colspan="2"></td><td>Total background events: 4</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
 
</td><td>Upwards: 1, 2, 8, 23, 29, 44, 46</td><td>Downwards: 0, 3, 4, 5, 12, 16, 19, 21, 27, 30, 33, 34, 35, 36, 37, 43, 48, 49</td></tr>
<tr><td>Total Upward muon events: 7</td><td>Total Downward muon events: 18</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
 
</td><td>Upwards: 6, 9, 11, 14, 15, 18, 20, 22, 32, 38, 39, 40, 42, 45</td><td>Downwards: 7, 10, 17, 24, 28, 31, 47</td></tr>
<tr><td>Total Upward electron events: 14</td><td>Total Downward electron events: 7</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{18}{7} = 2.6$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{7}{14} = 0.5$$

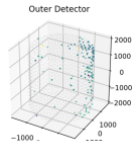
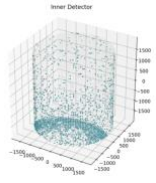
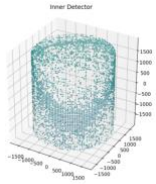
Results table – Data Set 3

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 6, 28, 30, 42, 48
(hits in outer detector)
 
</td></tr>
<tr><td colspan="2"></td><td>Total background events: 5</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
 
</td><td>Upwards: 0, 4, 5, 7, 8, 15, 20, 22, 31, 32, 43, 44, 46, 49</td><td>Downwards: 2, 11, 12, 16, 18, 24, 33, 35, 38, 39, 40, 45</td></tr>
<tr><td>Total Upward muon events: 14</td><td>Total Downward muon events: 12</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
 
</td><td>Upwards: 1, 3, 9, 10, 13, 14, 17, 19, 23, 25, 27, 29, 34, 37, 41</td><td>Downwards: 21, 26, 36, 47</td></tr>
<tr><td>Total Upward electron events: 15</td><td>Total Downward electron events: 4</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{12}{4} = 3.0$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{14}{15} = 0.9$$

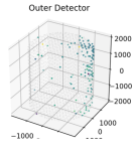
Results table – Data Set 4

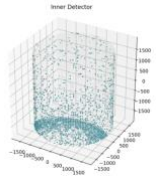
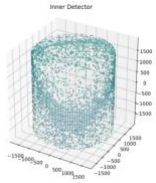
<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 6, 13, 16, 19, 27, 38, 44
(hits in outer detector)
 
</td></tr>
<tr><td colspan="3">Total background events: 7</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
  </td><td>Upwards: 1, 2, 5, 10, 12, 23, 29, 31, 37</td><td>Downwards: 15, 17, 24, 25, 26, 30, 32, 41, 42, 43, 45, 46, 49</td></tr>
<tr><td>Total Upward muon events: 9</td><td>Total Downward muon events: 13</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
  </td><td>Upwards: 0, 4, 7, 8, 9, 18, 22, 35, 36, 47</td><td>Downwards: 3, 11, 14, 20, 21, 28. 33, 34, 39, 40, 48</td></tr>
<tr><td>Total Upward electron events: 10</td><td>Total Downward electron events: 11</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{13}{11} = 1.2$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{9}{10} = 0.9$$

Results table – Data Set 5

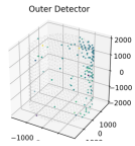
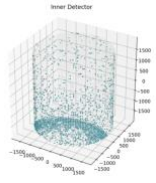
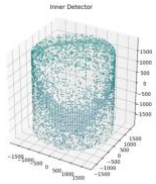
<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 7, 13, 44, 46, 47
(hits in outer detector)
 

Total background events: 5</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
 
</td><td>Upwards: 14, 20, 22, 26, 29, 36</td><td>Downwards: 1, 3, 5, 6, 8, 9, 10, 21, 25, 31, 34, 37, 39, 40, 42, 45</td></tr>
<tr><td>Total Upward muon events: 6</td><td>Total Downward muon events: 16</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
 
</td><td>Upwards: 0, 2, 12, 18, 19, 23, 27, 28, 35, 38, 41, 43, 48, 49</td><td>Downwards: 4, 11, 15, 16, 17, 24, 30, 32, 33</td></tr>
<tr><td>Total Upward electron events: 14</td><td>Total Downward electron events: 9</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{16}{9} = 1.8$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{6}{14} = 0.4$$

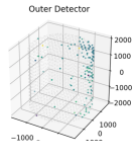
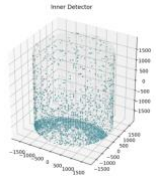
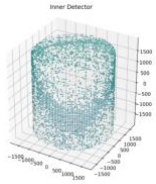
Results table – Data Set 6

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 7, 13, 25, 34, 47, 48
(hits in outer detector)
 
</td></tr>
<tr><td colspan="3">Total background events: 6</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
  </td><td>Upwards: 3, 19, 23, 31, 49</td><td>Downwards: 9, 10, 14, 15, 17, 20, 26, 27, 28, 32, 33, 35, 36, 37, 38, 39</td></tr>
<tr><td>Total Upward muon events: 5</td><td>Total Downward muon events: 16</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
  </td><td>Upwards: 0, 1,2, 4, 6, 8, 16, 18, 22, 29, 30, 43, 44, 45, 46, 49</td><td>Downwards: 5, 11, 12, 21, 24, 41, 42</td></tr>
<tr><td>Total Upward electron events: 16</td><td>Total Downward electron events: 7</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{16}{7} = 2.3$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{5}{16} = 0.3$$

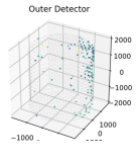
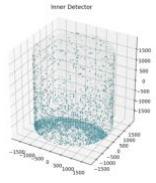
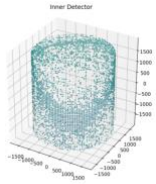
Results table – Data Set 7

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background:
(hits in outer detector)
 
</td></tr>
<tr><td colspan="2"></td><td>Total background events: 0</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
 
</td><td>Upwards: 3, 12, 13, 19, 23, 26, 33, 39, 44, 46, 48</td><td>Downwards: 2, 10, 11, 14, 17, 20, 22, 28, 29, 31, 35, 36, 41, 42, 43, 45, 49</td></tr>
<tr><td>Total Upward muon events: 11</td><td>Total Downward muon events: 17</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
 
</td><td>Upwards: 1, 4, 5, 8, 9, 15, 24, 25, 27, 30, 32, 38, 40, 47</td><td>Downwards: 0, 6, 7, 16, 18, 21, 34, 37</td></tr>
<tr><td>Total Upward electron events: 14</td><td>Total Downward electron events: 8</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{17}{8} = 2.1$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{11}{14} = 0.8$$

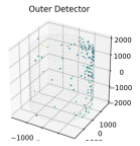
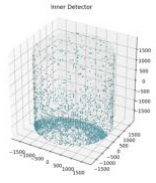
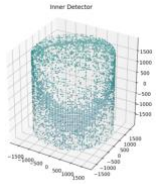
Results table – Data Set 8

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 47
(hits in outer detector)
 
</td></tr>
<tr><td colspan="3">Total background events: 1</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
 
</td><td>Upwards: 8, 9, 16, 19, 31, 39, 40, 42, 45, 49</td><td>Downwards: 7, 12, 13, 14, 18, 21, 23, 24, 25, 26, 28, 29, 46, 48</td></tr>
<tr><td>Total Upward muon events: 10</td><td>Total Downward muon events: 14</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
 
</td><td>Upwards: 0, 2, 5, 10, 11, 15, 27, 30, 32, 34, 35, 38, 43</td><td>Downwards: 1, 3, 4, 6, 17, 20, 22, 33, 36, 37, 41, 44</td></tr>
<tr><td>Total Upward electron events: 13</td><td>Total Downward electron events: 12</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{14}{12} = 1.2$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{10}{13} = 0.8$$

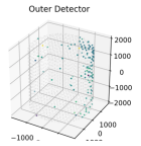
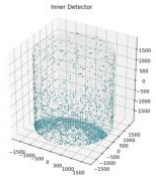
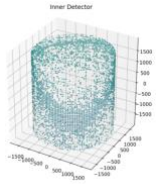
Results table – Data Set 9

<table>
<tr><td colspan="2">Event Range Analysed: 0-49</td><td>Total event number (N): 50</td></tr>
<tr><td colspan="3">Events classified as Background: 14, 15, 32, 43, 45
(hits in outer detector)
 
</td></tr>
<tr><td colspan="2"></td><td>Total background events: 5</td></tr>
<tr><td rowspan="2">Events Classified as muon-like (sharp ring)
 
</td><td>Upwards: 2, 5, 6, 12, 17, 20, 26, 30, 35, 38, 41, 46</td><td>Downwards: 4, 7, 9, 13, 18, 21, 22, 23, 24, 25, 27, 28, 33, 34, 36, 37, 39</td></tr>
<tr><td>Total Upward muon events: 12</td><td>Total Downward muon events: 17</td></tr>
<tr><td rowspan="2">Events classified as electron-like (fuzzy ring)
 
</td><td>Upwards: 3, 10, 11, 16, 31, 47, 48</td><td>Downwards: 0, 1, 8, 19, 29, 40, 42, 44, 49</td></tr>
<tr><td>Total Upward electron events: 7</td><td>Total Downward electron events: 9</td></tr>
</table>

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{17}{9} = 1.9$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{12}{7} = 1.7$$

Results table – Data Set 10

Event Range Analysed: 0-49		Total event number (N): 50
Events classified as Background: 10, 19, 20, 23, 24, 29, 49 (hits in outer detector) 		Total background events: 7
Events Classified as muon-like (sharp ring) 	Upwards: 4, 7, 12, 17, 27, 40, 42, 45, 46, 47	Downwards: 0, 2, 3, 8, 9, 11, 13, 18, 22, 25, 38, 43, 48
	Total Upward muon events: 10	Total Downward muon events: 13
Events classified as electron-like (fuzzy ring) 	Upwards: 1, 5, 6, 14, 21, 26, 28, 30, 31, 32, 33, 34, 35, 36, 39, 41	Downwards: 15, 16, 37, 44
	Total Upward electron events: 16	Total Downward electron events: 4

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{13}{4} = 3.25$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{10}{16} = 0.6$$

Step 5

Do your two ratios agree with what you expected?
Do your two ratios agree with each other?

The upwards ratio should be lower than the downwards ratio because upwards neutrinos have travelled further and have more time to oscillate. You should see that there are less upward-going muon events. The downward ratio should be close to the expected value of 2 but there are statistical fluctuations, especially if you have only analysed a small (eg <50) number of events.

Step 6

If you shared the task between groups – collect up the total number of events in each category in this table now:

Data Set:	1	2	3	4	5	6	7	8	9	10	Total
Upwards Muon	9	7	14	9	6	5	11	10	12	10	93
Downwards Muon	14	18	12	13	16	16	17	14	17	13	150
Upwards Electron	12	14	15	10	14	16	14	13	7	16	131
Downwards Electron	5	7	4	11	9	7	8	12	9	4	76
Background	10	4	5	7	5	6	0	1	5	7	50

Step 7

Add up the totals and record in the final column of the above table.

Calculate the total values for:

$$Downwards\ ratio = \frac{Number\ of\ downward\ muon\ events}{Number\ of\ downward\ electron\ events} = \frac{150}{76} = 2.0$$

$$Upwards\ ratio = \frac{Number\ of\ upward\ muon\ events}{Number\ of\ upward\ electron\ events} = \frac{93}{131} = 0.7$$

Conclusions:

- The Downwards ratio of muon to electron flavour is close to the expected value of 2 so we don't have any evidence for neutrino oscillation over the short path length from the atmosphere straight down into the detector
- The Upwards ratio is much smaller suggesting that muon type neutrinos have changed flavour, "oscillated" into electron type neutrinos on their longer journey all the way through the earth.

A few points to consider:

- We don't actually classify the events 'by eye' like this – physicists use computer algorithms to do this. You probably found that some events were not clear to classify and that your classifications differ from the official results given here.
- For a real analysis we'd need many more events – Super-K runs 24h/day and publishes results based on years of data. We measure a few thousand events over a couple of years.
- We have many more background events than in your data sets – over 99.99% of our data is background cosmic muons but that would be very boring for this exercise so we removed most of them! We see about 1 muon per second passing through Super-K!
- We have simplified things into just up and down for this exercise taking one path-length for upwards events but the length, L, depends on the direction of the track.

😎Well done for finishing!! Please let us know your thoughts about it using this form.