



King's International Foundation Programme

Self-Evaluation Mathematics Test

Instructions to students

1. Answer all the questions.
2. You should **NOT** use a calculator for this test.
3. A score lower than 70% indicates that you may benefit from improving your maths skills prior to starting at King's.
4. Time allowed: 1 hour

1. Work out the value of

a) $(6 \times 2) + (9 \div 3)$

b) $(7 \times 5) + 6 \times 8$

c) $6 + 12 \div 3$

d) $4 - (-3)$

e) $-6 + -9$

f) $36 \div -4 - 2$

g) $5^3 - (-2)^4$

h) $2^3 \times 3^2$

(8 marks)

2. Simplify the following expressions:

a) $4a \times 3a^2$

b) $2x^2 + x + x^2 - 3x - (-4x)$

c) $6(-x - 2) + 3(2x - 1)$

(5 marks)

3. Evaluate the following expressions

a) 4^0

b) $(-3)^3$

c) $81^{\frac{1}{4}}$

d) $\left| \frac{6}{5} \right|^{-2}$

(4 marks)

4. Show that $27^{\frac{1}{3}} \times 3^3 = 9^2$

(2 marks)

5. Write the number 12752

a) to the nearest 10

b) to the nearest 100

(2 marks)

6.

a) Find 20% of 90

b) Find $\frac{2}{5}$ of 30

(2 marks)

7. Solve the following equations

a) $4(2x-6)=5(3x+2)$

b) $\frac{2-4x}{5}=6$

c) $\frac{2x-4}{3} = \frac{-x-2}{2}$

d) $\frac{3}{2x} + \frac{1}{4} = 8$

(10 marks)

8. Factorise the following expressions

a) $4xy-10y$

b) $6x^3y^2+9xy$

c) x^2-5x+6

d) x^2-x-12

e) $4x^2-25y^4$

f) $4x^2 - 9x + 2$

(10 marks)

9. Calculate

a) $\frac{1}{4} + \frac{1}{3}$

b) $\frac{3}{8} \div \frac{1}{4}$

c) $\frac{4}{9} \times \frac{3}{2}$

(3 marks)

10. Solve the inequalities

a) $8x + 3 < 4x + 1$

b) $3(x + 7) \geq 2 + 4x$

(4 marks)

11. Simplify fully

a) $m^2 \times m^7$

b) $a^5 \div a^3$

c) $(x^3)^{12}$

d) $2x^2 y^3 \times 3xy^4$

(5 marks)

12.

a) Solve equation: $2x^2 + 5x - 3 = 0$

b) Solve inequality: $x^2 + x - 2 < 0$

(6 marks)

13. Depending on the value of parameter k discuss the nature of the roots of the following quadratic equation:

$$y = (k - 2)x^2 - (k + 1)x + 4 = 0$$

(8 marks)

14. Expand and simplify

a) $(3x-2)(x+5)$

b) $(4x+2)^2$

(4 marks)

15. Solve the simultaneous equations

$$8x - 2y = 16$$

$$4x - 6y = -12$$

(5 marks)

16. Sketch the following functions indicating, where appropriate, the coordinates of the points where the curve crosses the coordinate axes

a) $y = x^2$

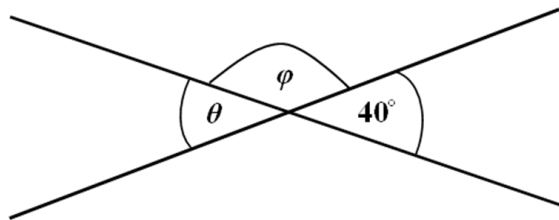
b) $y = \frac{1}{x}$

c) $y = x^3$

d) $y = e^x$

(8 marks)

17. The diagram shows two intersecting straight lines



a) Work out the value of angle θ .

b) Work out the value of angle φ .

(2 marks)

18. The point M has coordinates $(-2,5)$ and the point N has coordinates $(3,-4)$.

a) Find the gradient of MN .

b) Find the equation of the line which passes through points M and N . Give your answer in the form $ax+by+c=0$, where a , b and c are integers.

(4 marks)

19. Rationalise the denominator and simplify

$$\frac{3\sqrt{2} + \sqrt{6}}{3(\sqrt{2} + \sqrt{6})}$$

(4 marks)

20. Solve the following equation in the interval $-360^\circ \leq x \leq 360^\circ$

$$\sin x - \frac{1}{2} = 0$$

(4 marks)

END OF TEST

Total: 100 Marks

Answers to questions

1. a) 15 b) 83 c) 10 d) 7 e) -15 f) -11 g) 109 h) 72

(1 mark for each question)

2. a) $12a^3$ (1mark)

b) $3x^2 + 2x = x(3x + 2)$ (2 marks)

c) -15 (2 marks)

3. a) 1 b) -27 c) 3 d) $\frac{25}{36}$

(1 mark for each question)

4. $27^{\frac{1}{3}} \times 3^3 = 3 \times 3^3 = 3^4 = 9^2$ (2 marks)

5. a) 12750 b) 12800 (1 mark for each question)

6. a) 18 b) 12 (1 mark for each question)

7. Solve the following equations

a) $4(2x-6)=5(3x+2)$

$$8x - 24 = 15x + 10$$

$$-7x = 34 \quad \Rightarrow \quad x = -\frac{34}{7}$$

(2 marks)

b) $\frac{2-4x}{5}=6$

$$2-4x=30 \quad \Rightarrow \quad x=-7$$

(2 marks)

c) $\frac{2x-4}{3} = \frac{-x-2}{2}$

$$4x - 8 = -3x - 6$$

$$7x = 2 \quad \Rightarrow \quad x = \frac{2}{7}$$

(3 marks)

d) $\frac{3}{2x} + \frac{1}{4} = 8$

$$\frac{3}{2x} + \frac{1}{4} = 8 \quad / \times 4x$$

$$6 + x = 32x$$

$$31x = 6 \quad \Rightarrow \quad x = \frac{6}{31}$$

(3 marks)

8. Factorise the following expressions

a) $4xy-10y=2y(2x-5)$

(1 mark)

b) $6x^3y^2+9xy=3xy(2x^2y+3)$

(1 mark)

c) $x^2 - 5x + 6 = (x - 2)(x - 3)$

(2 marks)

d) $x^2 - x - 12 = (x-4)(x+3)$ (2 marks)

e) $4x^2 - 25y^4 = (2x-5y^2)(2x+5y^2)$ (2 marks)

f) $4x^2 - 9x + 2 = (x-2)(4x-1)$ (2 marks)

9. Calculate

a) $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ (1 mark)

b) $\frac{3}{8} \div \frac{1}{4} = \frac{3}{2}$ (1 mark)

c) $\frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$ (1 mark)

10. Solve the inequalities

a) $8x + 3 < 4x + 1$
 $4x < -2 \Rightarrow x < -\frac{1}{2}$ (2 marks)

b) $3(x + 7) \geq 2 + 4x$
 $3x + 21 \geq 2 + 4x \Rightarrow x \leq 19$ (2 marks)

11. Simplify fully

a) $m^2 \times m^7 = m^9$ (1 mark)

b) $a^5 \div a^3 = a^2$

(1 mark)

c) $(x^3)^{12} = x^{36}$

(1 mark)

d) $2x^2y^3 \times 3xy^4 = 6x^3y^7$

(2 marks)

12.

a) Solve equation: $2x^2 + 5x - 3 = 0$

$$x_{1,2} = \frac{-5 \pm \sqrt{25 - 4 \times 2 \times (-3)}}{4} \quad (1 \text{ mark})$$

$$x_{1,2} = \frac{-5 \pm 7}{4} \quad \Rightarrow \quad x_1 = \frac{1}{2} \quad \text{and} \quad x_2 = -3 \quad (2 \text{ marks})$$

b) Solve inequality: $x^2 + x - 2 < 0$

$$(x+2)(x-1) < 0 \quad (1 \text{ mark})$$

$$\Rightarrow -2 < x < 1 \quad (2 \text{ marks})$$

13. Depending on the value of parameter k discuss the nature of the roots of the following quadratic equation:

$$y = (k - 2)x^2 - (k + 1)x + 4 = 0$$

$$D = [(k + 1)]^2 - 4 \times (k - 2) \times 4$$

$$D = k^2 - 14k + 33 = (k - 3)(k - 11) \quad (2 \text{ marks})$$

If $k < 3 \cup k > 11$ roots are **real and different** (2 marks)

If $k = 3$ or $k = 11$ roots are **real and equal** (2 marks)

If $3 < k < 11$ **no real roots (roots are complex)**

(2 marks)

14. Expand and simplify

a) $(3x-2)(x+5)=3x^2+13x-10$ (2 marks)

b) $(4x+2)^2=16x^2+16x+4$ (2 marks)

15. Solve the simultaneous equations

$$8x - 2y = 16$$

$$4x - 6y = -12$$

$$8x - 2y = 16$$

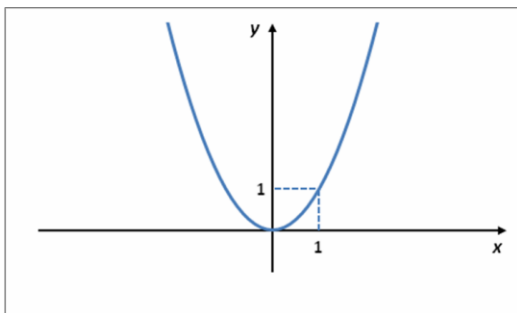
(subtract equations) $\Rightarrow 10y = 40 \Rightarrow y = 4$ (4 marks)

$$8x - 12y = -24$$

(substitute $y = 4$ in any of the two equations to get x) $\Rightarrow x = 3$ (1 mark)

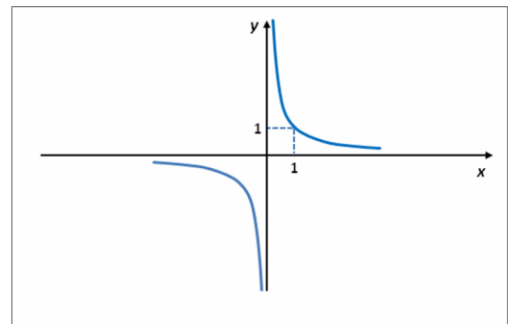
16. Sketch the following functions indicating, where appropriate, the coordinates of the points where the curve crosses the coordinate axes

a) $y = x^2$



(2 marks)

b) $y = \frac{1}{x}$

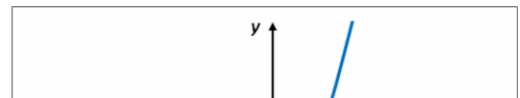


(2 marks)

c) $y = x^3$



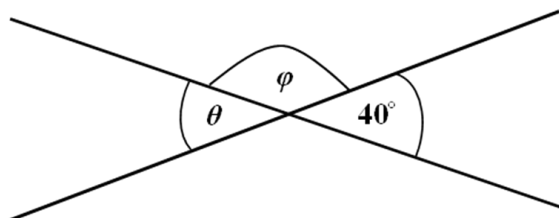
d) $y = e^x$



(2 marks)

(2 marks)

17. The diagram shows two intersecting straight lines



a) $\theta = 40^\circ$ (1 mark)

b) $\phi = 180^\circ - 40^\circ = 140^\circ$ (1 mark)

18. The point M has coordinates $(-2,5)$ and the point N has coordinates $(3,-4)$.

a) Find the gradient of MN .

$$m_{MN} = \frac{y_N - y_M}{x_N - x_M} = \frac{-4 - 5}{3 - (-2)} = \frac{-9}{5} \quad (1 \text{ mark})$$

b) Find the equation of the line which passes through points M and N . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

$$y - 5 = -\frac{9}{5}(x - (-2)) \Rightarrow 5y - 25 = -9x - 18 \Rightarrow 9x + 5y - 7 = 0$$

(3 marks)

19. Rationalise the denominator and simplify

$$\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{2}\sqrt{6}} = \frac{\sqrt{2}(3 + \sqrt{3})}{\sqrt{2}\sqrt{6}} = \frac{(3 + \sqrt{3})(1 - \sqrt{3})}{\sqrt{2}\sqrt{6}} = \frac{3 - 2\sqrt{3} - 3}{3(1-3)} = \frac{-2\sqrt{3}}{3 \times (-2)} = \frac{\sqrt{3}}{3} \quad (4 \text{ marks})$$

20. Solve the following equation in the interval $-360^\circ \leq x \leq 360^\circ$

$$\sin x - \frac{1}{2} = 0$$

$$\sin x = \frac{1}{2} \quad \Rightarrow \quad x = 30^\circ + n \times 360^\circ \quad \text{or} \quad x = 150^\circ + n \times 360^\circ \quad \text{where } n \in \mathbb{Z}$$

$$\Rightarrow x = -330^\circ, -210^\circ, 30^\circ, 150^\circ$$

(4 marks)